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Are Transition Countries Overbanked? The Effect of Institutions on Bank Market Entry

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General Equilibrium Model of an Economy with a Futures Market

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VORWORT

Mit dem hier vorgelegten Band der Veröffentlichungen von *forost* legt die Forschungsgruppe I zwei Arbeiten vor, die sich mit speziellen Problemen der Transformation unter Anwendung modelltheoretischer Ansätze auseinander setzen.

Roman Cech befasst sich mit der Frage, wie sich in einer Wirtschaft, die über Zukunftsmärkte verfügt und deren Akteure unterschiedliche Risikoaversionen besitzen, ein Gleichgewicht herausbildet. Dabei sind seine *Ergebnisse*, dass die Zukunftsmärkte die Entwicklung der realen Wirtschaft beeinflussen und auf die Einkommensverteilung in einer Volkswirtschaft Einfluss nehmen für die Entwicklung der Transformationsgesellschaften und ihre Annäherung an die EU von besonderer Bedeutung.

Christa Hainz überprüft die These, dass die Transformationsstaaten „overbanked“ seien, und sucht nach dem Grund für dieses Phänomen. Dabei zeigt sie, dass ein entscheidender Grund in den noch insgesamt unzureichend funktionierenden Institutionen während der Anfangphase der Transformation zu suchen ist. Daher ist zu erwarten, dass mit der Festigung der Institutionen auch eine Konsolidierung des Bankensystems einhergehen wird. Daher muss in allen Transformationsstaaten auch unter dem Gesichtspunkt der Konsolidierung des Bankensystems – was eine wichtige Voraussetzung für die weitere Entwicklung der Transformationsstaaten ist – der Festigung der Institutionen besondere Aufmerksamkeit gewidmet werden.

Beide Arbeiten sind von grundsätzlicher Bedeutung für der Annäherung der Transformationsstaaten an die EU und den in diesem Prozess zu erwartenden Entwicklungen, und sie erlauben Rückschlüsse auf die notwendigen wirtschaftspolitischen Maßnahmen.

München, im August 2003 Hermann Clement

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Are Transition Countries Overbanked? The Effect of Institutions on Bank Market Entry

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Abstract

The popular notion that transition countries are overbanked is challenged here. We study the decision for market entry and the optimal number of banks in a Salop-model. We show that the amount of collateral depends on the distance between bank and firm as well as the quality of the institutional environment. We analyze how the number of banks decreases as the institutional environment improves. Moreover, we find that market entry is insufficient because new entrants do not consider the positive effects of their entry decision on social welfare sufficiently, i.e. the reduction of collateral requirement and the increase in the average liquidation value.

JEL-Classification: D43, G21, G34, L13, P31, P34

Keywords: Transition economies, bank competition, market entry, corporate finance, corporate governance

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1. Introduction

The number of banks in transition countries soared in the beginning of the transition process. Russia is the most popular example: The number of banks peaked in 1996 at about 2,000, but it sunk significantly after the crisis in 1998 to about 1,300 in 2000 (EBRD, 2000; OECD, 2000). The strong decrease in the number of banks suggests that Russia was overbanked during the 1990s. Russia can be used to illustrate the situation in Eastern Europe where the banking sector is often described as overbanked but underserviced (Bonin et al., 1998). Which are the factors that render market entry so attractive?

To answer the questions on market entry we set up a theoretical model of spatial competition where banks have to bear the transportation costs. In transition countries banks collateralize nearly all credit contracts (Fan et al., 1996; Bratkowski et al., 2000). However, banks differ in their liquidation values of collateralized assets. Thus, the model provides new insights into the behavior of banks in transition countries. It also contributes to the theoretical literature on spatial competition. A traditional Salop-model with mill pricing shows that too many firms (or banks) enter.[1] In contrast, we use a more realistic model with delivered pricing since banks face a loss if collateralized assets are liquidated. The surprising result is that the economy is underbanked.

Due to the deficient legal and institutional environment in transition countries, the model has to portray the problems associated with collateralization. This field of problems is taken into account as follows: Banks have higher liquidation values of collateral if they are located closer to a firm. It is shown that collateralization is used to solve the moral hazard problem of finance. A bank makes a positive profit if it has a comparative advantage in liquidating collateralized assets of any particular firm. As banks cannot price-discriminate, they offer the same contract to all borrowers.

However, this analysis shows that market entry has interesting implications for the industry as a whole. As we would expect, market entry reduces the market share of each individual bank. At the same time, the amount of collateral and the repayment decrease as the distance between the firm and the bank is reduced. This is the negative externality which is typically found in a Salop-model. Interestingly, market entry increases the liquidation payoff, which is obtained from the assets of the marginal borrower. Therefore, it achieves a higher average return if assets have to be liquidated. However, the negative effects of market entry on bank profit dominate. And therefore the profit of each bank decreases as a new bank enters. From a social welfare perspective we find that the number of banks in equilibrium is too low. The banks do not internalize the positive effects of entry on social welfare. Market entry increases social welfare by reducing the costs of collateralization in two ways: first, by lowering the amount of collateral and second, by increasing the average liquidation value. Due to these positive externalities, the equilibrium number of banks is lower than the socially optimal number.

This paper is related to two areas in the literature: banking in transition countries and spatial competition. The empirical studies propose that transition countries are overbanked but underserviced (Bonin and Wachtel, 1999). Jaffee and Levonian (2001)

[1]The distinction between delivered and mill pricing is made by Gabszewicz and Thisse (1986).

carry out an empirical analysis of the number of banks in transition economies. They calculate a benchmark value for the number of banks in transition countries in 1995. To calculate the benchmark they use data on the number of banks in OECD countries in order to find out the main determinants of the structural characteristics of the banking system. In their calculation of the number of banks, GDP is the only significant variable.[2] However, the explanatory power is limited as the $R^2 = 0.617$ suggests. The authors conclude that other factors, such as regulation, have a strong influence on the number of banks. With regards to transition economies, they show that the number of banks is lower than the benchmark value in the Czech Republic, Hungary and Poland, whereas in all other countries it is higher.

The empirical literature on banking in transition economies also explores the effects of a deficient legal and institutional environment. McNulty and Harper (2001) find in their regression that these deficiencies are responsible for the low degree of financial intermediation. But how does the poor legal and institutional environment affect the banking sector? This relationship becomes most obvious under extremely widespread collateralization (Fan et al., 1996; Bratkowski et al., 2000). First, if banks rely on public contract enforcement they encounter several difficulties. In Russia, for instance, the bailiffs face an overwhelming caseload. Therefore, they carefully select the cases that they solve. As they usually have to travel to the location of a judgment creditor by public transportation, the incentive to solve cases in distant locations is low (Kahn, 2002). Second, as public contract enforcement often works insufficiently, some banks may prefer private contract enforcement (McMillan and Woodruff, 2000). This means that they have to bear significant costs for contract enforcement; moreover, they might have different costs. Finally, secondary markets do not function perfectly. Due to generally bad economic conditions, the demand for the assets is low and so are the returns for the bank. Since the institutional environment is poor, agents which have superior information or are integrated into networks can achieve a higher return.[3] The popularity of collateralization is surprising as the liquidation payoff for the bank is low. Nevertheless, collateralization helps solve the incentive problems associated with debt financing: adverse selection, moral hazard and state verification (Bester, 1985; Holmström, 1996; Bester, 1994).

Thus far, the theoretical literature on banking in transition countries has not studied the optimal number of banks intensively; it has largely been focused on non-performing loans and competition. However, the intensity of bank competition and the number of banks are related to each other. Schnitzer (1999) studies the bank's incentive to invest in perfect screening in a Salop-model. The analysis reveals that the incentive to invest in screening depends on the number of uninformed competitors. The screening costs determine whether all banks screen or no bank screens, and they also determine the extent of market entry. The first best number of banks enter only if screening costs are high. There are too many banks present in the case of low and intermediate screening costs. Generally, banks either screen too much from a social welfare point of view (if

[2] All other variables, i.e. population, size of the country, gross saving ratio, and ratio of non-resident claims to total claims on banks, are insignificant.

[3] Koford and Tschoegl (1999) provide evidence from Bulgaria.

screening costs are low) or they do not screen at all (if screening costs are intermediate).[4] Solely in the latter case is the banking sector overbanked but underserviced - as described in the transition literature.

The second area in the literature which this paper is complementary to is spatial competition. In his seminal paper, Salop (1979) develops a spatial model of market entry with mill pricing. From a normative point of view there are too many firms in the market. The case of a uniform delivered price is studied by Gromberg and Meyer (1981). They distinguish between extreme price competition and collusion among firms. However, they do not consider social welfare. To our knowledge, only the paper by Matsumura (2000) shows that too few firms enter. He considers the integer problem of the number of firms and shows that excess entry theorem occurs if the marginal production cost is constant. Whereas if the marginal production cost is increasing, the excess-entry theorem no longer holds true.

The paper is organized as follows: In Section 2, the Salop model and the optimal credit contracts are studied. In Section 3, we first analyze market entry into the banking sector and then compare the number of banks in equilibrium with the socially optimal number. Finally, in Section 4, the results are discussed by relating them to the empirical evidence.

2. Bank Competition and Asset Specificity

2.1. Model

In this model firms which want to undertake an investment project with costs of I are studied. Each firm has an asset endowment of A. Firms can only realize the project if they receive a credit because they do not have sufficient liquid means. The expected return of the project depends on the effort of the firm's manager. If effort E is exerted, the probability for the successful outcome with a high return of X increases from p_L to p_H. In the case of failure, the return is 0. Firms are uniformly distributed along a circular road of length 1 and their total mass is normalized to 1.

In a first best environment, the bank observes the effort level of the manager and, as we assume that $p_H X - E > I > p_L X$, the contract determines that the manager exerts effort E. However, as the effort is not observable in reality, the credit contract has to be designed in a manner that gives the firm's manager an incentive to exert effort. The manager's incentive depends on the state of the world where the return is determined by the gross return and the repayment to the bank. The firm has to repay R in the case of success. If the project fails, the amount of collateral L is liquidated by the bank. It is assumed that the asset endowment is high enough to avoid problems of insufficient collateralizable wealth, i.e. $A \geq \frac{E}{p_H - p_L} p_L$.[5]

[4] A different set up is chosen by Broecker (1990): He studies a model with imperfect but costless screening. It is shown that the higher the number of banks, the higher is the repayment as the average quality of the applying firm decreases.

[5] A monopolistic bank demands collateral of $\frac{E}{p_H - p_L} p_L$ (see Hainz, 2003). Holmström (1996) studies

The banking sector consists of N banks. Each bank is allowed to locate in only one location. The banks do not choose their location, they are automatically located equidistantly from one another. In this way maximal differentiation between banks is exogenously imposed. The banks compete in Bertrand fashion for firms which are asking for credit.[6] Banks as well as firms are risk-neutral. Due to the substantial costs in the case of liquidation, the return to the bank, denoted by α, is lower than the continuation value within the firm, i.e. $0 \leq \alpha \leq 1$. In transition economies, liquidation leads to a considerable discount because public as well as private contract enforcement is expensive and secondary markets function only very imperfectly.

The time structure of the game is summarized in the following figure:

Figure 1: Time Structure

In the following section, we study the design of a credit contract for a given number of competitors in the banking sector. In the next step, the number of banks will be determined endogenously.

2.2. Bank Competition and Collateralized Credit Contracts

The bank's liquidation payoffs of the collateralized assets depend on the distance between the location of the bank and the firm. Thus, α is higher for firms in close proximity to a bank. To keep the analysis as simple as possible, we do not consider transaction costs and set $\alpha = 1$ for firms located at the same place as a bank. The further the distance which a firm has to travel to a bank, the lower is α. It is assumed that α decreases proportionally with the distance travelled. The liquidation value is lowest for a firm that is located directly opposite the bank on the circle, and is denoted by $\underline{\alpha}$. The liquidation value of the marginal firm is denoted by α^K.

There are two possible interpretations of what termed "distance" in our model. First, if there is spatial competition, it is the physical distance that causes costs.[7] For the bank closest to a firm the costs of enforcing the contract are lower than that of any other competitor. Second, banks specialize in financing firms from certain sectors. In that case,

the problems which are caused if firms do not possess sufficient collateral.

[6]The costs of funds are normalized to zero. In this analysis we focus on bank competition on credit markets. For transition countries Dittus and Prowse (1996) show that only a few banks take deposits, which they transfer to the credit-granting banks through the money market.

[7]There is evidence from Belgium for spatial competition in the credit market (Degryse and Ongena, 2002).

the bank that is perfectly specialized in the business of the firm has the best expertise in liquidating the assets because it knows the market and its players best. If they finance firms that are not belonging to their core group of customers, banks have higher costs translating into a lower liquidation value. This is particularly important in transition economies since the secondary markets work poorly and therefore the expertise of each individual bank plays a crucial role.

Furthermore, we assume that the bank cannot observe the location of each individual firm that is asking for credit and that firms ask for credit at the closest bank if the banks offer identical credit contracts. This assumption rules out price discrimination. Although the relative size of the different effects changes, price discrimination would not change the insights gained from this analysis. The assumptions can be justified as follows: In the case of spatial competition, public contract enforcement renders predicting the liquidation value of a particular asset difficult, as the example Russia shows. The incentives of the bailiffs to solve a case depend on the various features of a case, one of which is distance. Their incentives to solve a particular case should decrease the further away the judgment creditor is located. However, there might be economics of scale if various cases have to be solved at a particular location. The banks knowledge is limited to information pertaining to their own customers. Hence, firms and bailiffs should be better informed than banks about disputes of other firms in their surrounding area and therefore a more precise assessment of the liquidation value. Moreover, banks that enter the market do not know the markets for collateralized assets very well. Firms are better informed than banks with respect to the value of the collateralized asset in the case of failure.

The objective of the representative bank i is to maximize profit Π_i^B given by:

$$\max_{R,L} \; \Pi_i^B = \frac{1}{N}\left(\int_{\alpha^K}^{1} \left(p_H R + (1-p_H)\,\alpha L - I\right) d\alpha\right) \tag{2.1}$$

The bank's profit depends on its market share, given by $\frac{1}{N}$, and its profit from the firms financed, given by $\int_{\alpha^K}^{1} \left(p_H R + (1-p_H)\,\alpha L - I\right) d\alpha$. Customers are located on the right and the left hand side of the bank. The bank serves all firms located on the circle between the marginal firm, having a liquidation value of α^K, and the firm at its own location, having a liquidation value of 1. The liquidation value for the marginal firm, $\alpha^K = \left(1 - \frac{(1-\underline{\alpha})}{N}\right)$, depends on the lowest liquidation value and the number of banks. The latter determines the distance between the marginal firm and the bank. The liquidation value is lowest, i.e. $\alpha = \underline{\alpha}$, if a firm is financed by the bank that is located opposite on the circle.

When determining the terms of the contract $\{R, L\}$ the bank has to take into account the following constraints: First, the credit contract has to provide sufficient incentives to the firm's manager to exert effort; failing which, the bank's expected profit is negative (**?). The firm's incentive compatibility constraint is defined by:

$$\begin{aligned} & p_H\,(A + X - R) + (1-p_H)\,(A - L) - E \\ \geq \;\; & p_L\,(A + X - R) + (1-p_L)\,(A - L) \end{aligned} \tag{IC-F}$$

$$R \leq X + L - \frac{E}{\Delta p}$$

with $\Delta p = (p_H - p_L)$. We assume that $0 > p_H X - I - \frac{E p_H}{\Delta p} \geq -\frac{E p_L}{\Delta p}\left(p_H + (1 - p_H)\,\alpha^K\right)$, as (a) it is not possible to solve the problem without collateralization and (b) the payoff of investment is high enough to cover the costs associated with collateralization.[8]

Second, the firm has to be at least as well off with the credit financed investment as without it. The firm's participation constraint is given by:

$$p_H\,(A + X - R) + (1 - p_H)\,(A - L) - E \geq A \qquad \text{(PC-F)}$$

And finally, the bank has to consider the offers of the competing banks. The credit contract, which results from the maximizing the bank's profit function, subject to the above-mentioned constraints, is described in Proposition 1:

Proposition 1. *All banks offer the same credit contract which determines the repayment and the amount of collateral, i.e.* $R = \frac{\alpha^K (1-p_H)\left(X - \frac{E}{\Delta p}\right) + I}{p_H + (1-p_H)\alpha^K}$ *and* $L = \frac{-p_H X + \frac{E}{\Delta p} p_H + I}{p_H + (1-p_H)\alpha^K}$.

Proof: See Appendix.

By demanding a collateral the firm's payoff in the case of failure is reduced, the difference between state-contingent payoffs increases and thus the incentive of the firm's manager to exert effort increases. As (IC-F) shows, the collateral requirement L also influences the repayment R, which can be demanded without destroying the manager's incentive. The amount of collateral is determined by the liquidation value of the marginal firm's assets, α^K. The marginal firm is located half-way between two banks. For this firm, the banks engage in perfect competition as their liquidation value assessment for this firm is the same. The marginal firm has to generate an expected profit of zero for each bank, i.e. $p_H\left(X + L - \frac{E}{\Delta p}\right) + (1 - p_H)\,\alpha^K L - I = 0$. As it is not possible to discriminate between firms from different locations, the bank demands the same collateral from all firms. The assets of the other firms, which are located closer to the bank, have a higher liquidation value, and therefore these firms yield a positive profit for the bank.[9]

Offering the credit contract described in Proposition 1, the bank's profit is:

$$\Pi_i^B = \frac{1}{N}\left(p_H R + (1 - p_H)\left(\frac{1 + \alpha^K}{2}\right) L - I\right) \qquad (2.2)$$

inserting $R = X + L - \frac{E}{\Delta p}$ and $L = \frac{-p_H X + \frac{E}{\Delta p} p_H + I}{p_H + (1-p_H)\alpha^K}$ yields

$$\Pi_i^B = \frac{1}{N}\left(-p_H X + I + \frac{E}{\Delta p} p_H\right)\frac{(1 - p_H)\left(1 - \alpha^K\right)}{2\left(p_H + (1 - p_H)\,\alpha^K\right)}$$

[8] Formally, the first assumption implies that the bank's zero profit constraint after inserting (PC-F) would not hold if $L = 0$. The second assumption affirms that the firm's participation constraint holds for the optimal L determined in Proposition 1.

[9] If the banks knew where a firm is located, the collateral would be higher for all firms except the marginal firm. The explanation is as follows: Bank B which is further away from the firm is less specialized than bank A which is closest. Thus, bank B needs a higher repayment to make zero profit. Accordingly, bank B can demand a higher repayment than in the case without price discrimination, and it can extract more rent.

3. Market entry into the banking sector

3.1. Individual Entry Decision

So far the number of banks has been taken as given. What does a change in the number of competing banks and the resulting change in the degree of bank competition mean for the firms? The effect of changing the number of banks is summarized in the following Proposition.[10]

Proposition 2. *The greater the number of banks that compete in the banking sector, the lower is the repayment and the amount of collateral, i.e.* $\frac{\partial R}{\partial N} < 0$, $\frac{\partial L}{\partial N} < 0$.

Proof: See Appendix.

Technically, a higher number of banks means that the distance which the firms have to travel to their bank decreases. Consequently, their assets have a higher liquidation value and the banks need less collateral to fulfill the break-even condition for the marginal borrower. A lower collateral requirement also reduces the repayment and increases the net return for the investing firm. However, studying the effect of market entry on bank profit, we discover a new type of externality of the entry decision of an individual bank. This is expounded in the following Proposition.

Proposition 3. *Market entry by one bank has not only a negative but also a positive externality on the return of all other banks. A positive externality arise because the average liquidation value increases as the payoff obtained from liquidating the collateralized assets of the marginal debtor increases, i.e.* $\frac{\partial \alpha^K}{\partial N} > 0$.

Proof: See Appendix.

In our model, market entry by one bank has three different effects on all other banks. First, the market share of each bank decreases. The effect on the profit of each individual bank is negative, i.e. $-\frac{1}{N^2}\left(-p_H X + I + \frac{E}{\Delta p} p_H\right) \frac{(1-p_H)(1-\underline{\alpha})}{2(N-(1-p_H)(1-\underline{\alpha}))} < 0$. Second, the collateral requirement and the repayment decrease. With a higher number of banks, the distance between the marginal firm and the bank decreases. Therefore, the liquidation value of the assets from the marginal firm increases; and thus, the banks need less collateral in order to make zero profit. The effect of the lower collateral requirement and the lower repayment on the profit of each bank is negative, i.e. $-\frac{1}{N^2}\left(-p_H X + I + \frac{E}{\Delta p} p_H\right) \frac{(1-p_H)(1-\underline{\alpha})(2N-(1-p_H)(1-\underline{\alpha}))}{2(N-(1-p_H)(1-\underline{\alpha}))^2} < 0$. These two effects are negative externalities and are also found in a standard Salop-model. Third, in our model there is also a positive externality. Market entry by one bank increases the average liquidation value for all banks because the distance between the bank and the firm decreases. Due to this effect, the profit of each bank increases, i.e. $\frac{1}{N^2}\left(-p_H X + I + \frac{E}{\Delta p} p_H\right) \frac{(1-p_H)(1-\underline{\alpha})}{2(N-(1-p_H)(1-\underline{\alpha}))} > 0$. Altogether, the negative externalities outweigh the positive externality. Consequently, market entry by one bank decreases the profit of each incumbent bank.[11]

[10] Using comparative statics, we can compare different banking sectors without explicitly studying the bank's entry and exit decision.

[11] This result is not due to the specific functional form of our model. It would also be obtained if the liquidation payoff of each unit of collateral was, for instance, α^2.

Although the results in Proposition 3 are independent of the functional form chosen, it has one particularity. The negative effect of the decreasing market share on profit is just offset by the positive effect of the higher liquidation value. This is due to the fact that the change in the liquidation value is equal to the change in distance between bank and firm, which we use to measure the market share. However, it can easily be shown that the negative effects still dominate if the link between the change in market share and change in liquidation values remains linear but differs from one (as chosen in our model). To observe this, the effect that market entry has on profits by increasing the liquidation value and by decreasing the collateral requirement, as well as the repayment, are compared. The net effect of these two opposing effects is negative.

In a dynamic world the number of banks is not fixed. Potential bankers consider whether it is profitable to enter the banking sector. To establish a bank, the investor has to incur fixed costs F that include, for example, the expenditure for equipment such as computers. An investor decides to enter if

$$\Pi_i^B = \frac{1}{N}\left(-p_H X + I + \frac{E}{\Delta p} p_H\right)\frac{(1-p_H)\left(1-\alpha^K\right)}{2\left(p_H + (1-p_H)\,\alpha^K\right)} \geq F \tag{3.1}$$

The institutional environment in transition economies is still imperfect. In the context of this model this plays a role as the costs of contract enforcement and the imperfect secondary markets influence the liquidation value of collateral α and therefore the bank's profit Π_i^B. In our model, this can be incorporated by a parameter t for the quality of the institutional environment, with $\frac{\partial \alpha}{\partial t} > 0$.

Comparative statics provide insight into the optimal number of banks (ignoring integer problems) in different institutional environments as the following Proposition shows:

Proposition 4. *In a more developed institutional environment, the equilibrium number of competing banks is lower, i.e.* $\frac{\partial N}{\partial t} < 0$.

Proof: See Appendix.

A better institutional environment means that the average liquidation value increases. This, in turn, increases bank profit. The effect of a higher average liquidation value was discusses as the third effect above. Previously, the effect was due to market entry. Qualitatively, the same result is obtained if institutions improves as $\alpha^K = \left(1 - \frac{(1-\alpha)}{N}\right)$. However, the higher liquidation value implies that the collateral requirement decreases and consequently, the repayment is lower as well - both effects decrease profit. The net effect on bank profit is negative because the impact of the lower collateral requirement/ repayment dominates as argued above. Thus, the expected profit of an entering bank is lower. Market entry is less attractive and the number of banks that enter in equilibrium is lower.

This result has interesting implications for the differences in the banking sectors in transition economies. First of all, we can explain why the number of banks varies strongly among different countries. Our model predicts that in countries with a poor institutional environment, the equilibrium number of banks is higher than in countries where the institutional environment is more developed. Second, the equilibrium number of banks should decrease in the process of transition as the quality of the institutional environment improves.

3.2. Optimal Number of Banks

The analysis so far has explained the bank's incentive to enter the market. Market entry of an additional bank has negative effects on the profit of all other banks because their market share decreases as well as their market power, which allows them to extract rents. On the other hand, market entry by one bank has a positive externality on all other banks because the average liquidation value of collateral increases. To provide an answer to the question whether transition countries are overbanked we have to study the socially optimal number of banks.

Suppose that a benevolent social planner determines the optimal number of banks N^* by maximizing the following welfare function, consisting of the net return of investment less the costs of the banking system:

$$\max_{N^*} SW = p_H X - I - E - \underbrace{\left(\frac{\left(-p_H X + I + \frac{E}{\Delta p} p_H\right)}{\left(p_H + (1-p_H)\,\alpha^K\right)} (1-p_H) \left(1 - \left(\frac{1+\alpha^K}{2}\right)\right) + NF \right)}_{\text{costs of the banking system}} \tag{3.2}$$

Maximizing social welfare is equivalent to minimizing the costs of the banking system, which consists of the loss caused by the liquidation of collateral and the fixed costs for market entry. The following Proposition compares the number of banks that enter in equilibrium with the socially optimal number of banks:

Proposition 5. *Transition countries are underbanked, i.e.* $N^* > N$.

Proof: The following first order condition determines the optimal number of banks:

$$\frac{\left(-p_H X + I + \frac{E}{\Delta p} p_H\right)(1-p_H)(1-\underline{\alpha})}{2\left(N - (1-p_H)(1-\underline{\alpha})\right)^2} = F \tag{3.3}$$

Thus, the marginal increase in social welfare due to the lower loss associated with collateralization is equal to the fixed costs of market entry.

Market entry will occur until the marginal bank faces the following condition:

$$\frac{\left(-p_H X + I + \frac{E}{\Delta p} p_H\right)(1-p_H)(1-\underline{\alpha})}{2N\left(N - (1-p_H)(1-\underline{\alpha})\right)} = F \tag{3.4}$$

Accordingly, banks enter until the average expected profit is equal to the fixed costs of market entry.

For a given number of banks the difference between equation (3.3) and (3.4) is given by:

$$\frac{\left(-p_H X + I + \frac{E}{\Delta p} p_H\right)(1-p_H)(1-\underline{\alpha})}{2\left(N-(1-p_H)(1-\underline{\alpha})\right)^2} - \frac{\left(-p_H X + I + \frac{E}{\Delta p} p_H\right)(1-p_H)(1-\underline{\alpha})}{2N\left(N-(1-p_H)(1-\underline{\alpha})\right)}$$

$$= \frac{\left(-p_H X + I + \frac{E}{\Delta p} p_H\right)(1-p_H)(1-\underline{\alpha})}{2N\left(N-(1-p_H)(1-\underline{\alpha})\right)^2} > 0$$

As the effect of market entry on social welfare is higher than on the individual bank's profit and as both functions are concave, the socially optimal number of banks, N^*, is higher than the number of banks entering in equilibrium, N. Therefore, transition countries are underbanked. Q.E.D.

There are two reasons why the individual decision for market entry diverges from the social choice. First, the bank's decision about market entry is orientated towards the average profit to be expected. In contrast, the social choice is determined by the marginal effect of market entry on social welfare. Second, the social welfare function differs from the profit function. In our model, social welfare is reduced by the costs of collateralization. These costs decrease when the average loss of liquidation decreases and when the amount of collateral decreases. As market entry reduces the collateral requirement and the average loss of liquidation, entry influences social welfare positively through both "channels". And these positive influences on social welfare are the reason why a social planner wants a higher number of banks. Compared to the social planner's decision, the individual bank neglects the positive effects of its decision on social welfare. First, market entry by one bank increases the liquidation value for all banks, and thereby social welfare. However, the entrant bank considers only the positive effect on its own profit. Second, market entry reduces the amount of collateral, which reduces the costs of collateralization and thereby increases social welfare. However, for the entrant bank less collateral decreases its expected profit. As a consequence, entry remains insufficient.[12]

4. Discussion and Conclusion

The prediction of our model is that the number of banks decreases as the institutional environment improves. What has been observed in transition countries is that the number of banks soared in the first years of transition but slightly decreased later (EBRD, 2002). The decrease observed recently might be explained by the better institutions that limit the bank's scope for extracting rents.

The other important result of our model is that transition countries are underbanked. The evidence from Jaffee and Levonian (2001) is that transition countries can be overbanked as well as underbanked (see Table 1 in the Appendix). According to their calculation the Czech Republic, Hungary and Poland are underbanked. Whereas all other

[12] Matsumura (2000) shows that the excess-entry theorem does not always hold if the integer problem is considered and marginal production cost is increasing. He warns against using the excess entry theorem precipitately, i.e. its application in the Japanese Large-Scale Retail Act which restricts the new entry of retailers.

countries, among them Russia, are overbanked. How can we explain the difference between the result of Jaffee and Levonian and our theoretical prediction? First of all, their benchmark calculation depends only on the level of GDP, neglecting other important factors such as the institutional environment. The socially optimal number of banks increases as institutions deteriorate. Thus, their benchmark should be lower than the socially optimal number of banks of our analysis. Second, the fixed costs that banks face might be lower than the actual fixed costs of establishing a bank. In Russia, for instance, many banks are founded by firms. These so-called pocket banks get access to premises or office facilities more easily and more cheaply because they are leased below market prices. Therefore, their costs of market entry are lower than the actual costs, which are considered by a social planner. Under such circumstances, the incentive for market entry increases. The gap between the equilibrium and the optimal number of banks decreases as the institutions improve.

What are the effects of underbanking? We have argued that a lower number of banks increases the collateral requirement. We did not explicitly model credit rationing. However, it is obvious that some firms would lose access to financing if the collateral requirement becomes more demanding. Moreover, we should take into account that information about the asset endowment is asymmetric. A bank's reaction to the lack of information could be credit rationing which increases with a higher collateral requirement (Hainz, 2003). The negative consequences of underbanking can be reduced if the institutional environment improves. Therefore, the following measures should be considered. First, the laws on collateralization must be drafted carefully and unambiguously. Second, public contract enforcement has to become cheaper and more reliable. This would reduce the incentive for private contract enforcement; under which a bank can extract rents since it possesses a better contract enforcement technology than its competitors. Finally, the development of secondary markets has to be fostered. Improving the legal framework not only reduces the costs of collateralization by facilitating its seizure but also by increasing the efficiency of secondary markets.

5. Appendix

5.1. Proof of Proposition 1

(1) The firm
In our model the banks cannot price-discriminate. Therefore, we have to find the contract which specifies the most favorable contract terms $\{R, L\}$ for the marginal firm.

(2) The bank i
We first specify the contract for the marginal firm that yields zero profit for the bank. We then show that it is optimal to offer this contract.

As we have assumed that the banks are equidistantly distributed, and α_i decreases proportionally with the distance travelled, the marginal firm where $\alpha_i = \alpha_j \left(= \alpha^K\right)$ is located midway between the two banks. Due to (IC-F) and $\alpha^K \leq 1$, bank i sets $R = X + L - \frac{E}{\Delta p}$. From the bank's zero profit constraint of the marginal firm:

$$\Pi_i^B = p_H \left(X + L - \frac{E}{\Delta p}\right) + (1 - p_H)\,\alpha^K L - I = 0, \tag{5.1}$$

the optimal collateral is obtained $L = \frac{\left(-p_H X + I + \frac{E}{\Delta p} p_H\right)}{p_H + (1-p_H)\alpha^K}$. According to (IC-F) $R = \frac{\left(X - \frac{E}{\Delta p}\right)(1-p_H)\alpha^K + I}{p_H + (1-p_H)\alpha^K}$. Due the assumptions on the size of A, (PC-F) is fulfilled.

If both banks offer the same credit contract as specified above, they have the same market share, the profit from the marginal firm is zero and the total profit of each bank is positive. Deviating from this contract reduces a bank's profit. On the one hand, bank i does not have an incentive to increase the collateral requirement and the repayment because it would lose all its customers and make zero profit. On the other hand, bank i does not have an incentive to decrease the collateral requirement and the repayment because it would attract firms that are further away than the marginal firm. Profit generated from the marginal firm profit is zero. Thus, the profit generated from firms that are located further away would be negative. Q.E.D.

5.2. Proof of Proposition 2

$\frac{\partial L}{\partial N} = -\left(-p_H X + I + \frac{E}{\Delta p} p_H\right)(1 - p_H)\frac{1-\underline{\alpha}}{(N-(1-p_H)(1-\underline{\alpha}))^2} < 0$

$\frac{\partial R}{\partial N} = -\left(-p_H X + I + \frac{E}{\Delta p} p_H\right)(1 - p_H)\frac{1-\underline{\alpha}}{\Delta p(N-(1-p_H)(1-\underline{\alpha}))^2} < 0$ Q.E.D.

5.3. Proof of Proposition 3

$\frac{d\Pi_i^B}{dN} = -\left(-p_H X + I + \frac{Ep_H}{\Delta p}\right)(1-p_H)(1-\underline{\alpha})\frac{2N-(1-p_H)(1-\underline{\alpha})}{2N^2(N-(1-p_H)(1-\underline{\alpha}))^2} < 0$

To determine the sign of $2N - (1-\underline{\alpha})(1-p_H)$ we have to determine the equilibrium number of banks. Bank decide to enter until

$$\left(\frac{\left(-p_H X+I+\frac{Ep_H}{\Delta p}\right)}{N}\right)\frac{(1-p_H)\left(1-\left(1-\frac{(1-\underline{\alpha})}{N}\right)\right)}{2\left(p_H+(1-p_H)\left(1-\frac{(1-\underline{\alpha})}{N}\right)\right)} - F = 0.$$

Thus, the equilibrium number of banks in the market is:

$$\begin{aligned}
N_1 &= \frac{1}{2F}(1-p_H)(1-\underline{\alpha})F + \\
&\quad \frac{1}{2F}\sqrt{(1-p_H)(1-\underline{\alpha})F\left((1-p_H)(1-\underline{\alpha})F + 2\left(-p_H X + I + \frac{Ep_H}{\Delta p}\right)\right)} \\
N_2 &= \frac{1}{2F}(1-p_H)(1-\underline{\alpha})F - \\
&\quad \frac{1}{2F}\sqrt{(1-p_H)(1-\underline{\alpha})F\left((1-p_H)(1-\underline{\alpha})F + 2\left(-p_H X + I + \frac{Ep_H}{\Delta p}\right)\right)}
\end{aligned}$$

where N_2 can be excluded because it would be negative.

Therefore, we conclude that $2N - (1-\underline{\alpha})(1-p_H) > 0$.

The total effect of market entry on bank profit consists of the following effects:

(1) increasing liquidation value α^K (for given market share, repayment and collateral):

$\left(\frac{1}{N^2}\left(-p_H X + I + \frac{Ep_H}{\Delta p}\right)\frac{(1-p_H)(1-\underline{\alpha})}{2(N-(1-p_H)(1-\underline{\alpha}))}\right) > 0$

(2) decreasing collateral L and repayment R (for given market share and liquidation value):

$\left(-\frac{1}{N^2}\left(-p_H X + I + \frac{Ep_H}{\Delta p}\right)\frac{(1-p_H)(1-\underline{\alpha})(2N-(1-p_H)(1-\underline{\alpha}))}{2(N-(1-p_H)(1-\underline{\alpha}))^2}\right) < 0$

(3) decreasing market share (for given liquidation value, repayment and collateral):

$\left(-\frac{1}{N^2}\left(-p_H X + I + \frac{Ep_H}{\Delta p}\right)\frac{(1-p_H)(1-\underline{\alpha})}{2(N-(1-p_H)(1-\underline{\alpha}))}\right) < 0$ Q.E.D.

5.4. Proof of Proposition 4

Totally differentiating Π_i^B we obtain

$$\frac{dN}{dt} = -\frac{\frac{d\Pi_i^B}{dt}}{\frac{d\Pi_i^B}{dN}} < 0 \text{ as}$$

$$\frac{d\Pi_i^B}{dt} = -\left(-p_H X + I + \frac{Ep_H}{\Delta p}\right)\frac{(1-p_H)\frac{\partial\underline{\alpha}}{\partial t}}{2(N-(1-p_H)(1-\underline{\alpha}))^2} < 0$$

$$\frac{d\Pi_i^B}{dN} = -\left(-p_H X + I + \frac{Ep_H}{\Delta p}\right)(1-p_H)(1-\underline{\alpha})\frac{2N-(1-p_H)(1-\underline{\alpha})}{2N^{2(N-(1-p_H)(1-\underline{\alpha}))2}} < 0$$ Q.E.D.

5.5. Table 1

Table 1: Analyzing The Number of Banks in Transition Economies

	Actual Number	Benchmark Number	GDP per Capita	EBRD Legal Transition Indicator
Poland	81	82	3487	4
Czech Republic	51	55	4885	4
Slovak Republic	29	25	3443	3
Slovenia	29	22	10499	3
Lithuania	12	7	2434	3
Hungary	43	62	6019	4
Belarus	38	21	2329	2
Ukraine	188	101	3042	2
Uzbekistan	29	13	1014	2
Bulgaria	47	17	2305	3
Estonia	14	5	3055	2
Albania	9	3	1751	2
Kazakstan	101	29	1963	2
Mongolia	13	3	911	2
Krygz Republic	18	4	1746	2
Latvia	33	7	3707	3
Russia	2030	367	3983	3
Croatia	60	9	4266	4
Armenia	33	3	1425	3
Georgia	61	3	1151	2
Azerbaijan	136	5	1174	1

Source: Jaffee and Levonian (2001), p. 169.

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General Equilibrium Model Of An Economy With A Futures Market

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Abstract

General equilibrium effects of a futures market are analyzed in a two-sector model of an economy populated with agents who have differential risk aversion. Conditions leading to changes in the industry formation are derived and their effect on agents' welfare is measured by equivalent variation. A class of speculators who most benefit from the futures market endogenously arises in equilibrium.

1 Introduction

This paper presents a general equilibrium model of an economy with futures markets, in which futures trading arises among agents whose only heterogeneity is their degree of risk aversion. The economy produces two goods. The production is deterministic in one industry, but it is random in the other. Agents specialize in the production of one of the two goods. While all agents are exposed to the same price risk as consumers of the two goods, they are exposed to a different, industry-specific, income risk as producers. They choose which industry to enter based on the income risk. When given the option of sharing risk through the futures market, agents trade futures for two reasons: because they have a differential risk aversion, and because they have differential income risk exposure.

The General Equilibrium pricing of derivative securities, including futures contracts, is well understood in two cases: when there exists a complete set of contingent claims markets (Arrow, 1953), or when agents are identical (Lucas, 1978). Unfortunately, these models are too simple to study many aspects of futures trading, because real world asset markets are incomplete and agents are not identical. By contrast, in the model presented here, agents differ in their risk aversion and risk exposure, and asset markets are incomplete.

In spirit, this model is an extension of Kihlstrom and Laffont (1979). In that model, there exists a continuum of agents differing only in their degree of risk aversion who decide whether to become entrepreneurs or workers. Our model, like that of Kihlstrom and Laffont, uses a continuum of agents with differential risk aversion to explore industry choice and futures trading.

It seems natural to link the agents' industry choice and their behavior in the futures market to their risk aversion. Intuitively, risk sharing is what futures trading is all about. Yet, most of the literature on the topic is based on partial equilibrium analysis, with little, if any, role for risk sharing.[1] The few general equilibrium models of futures markets do not shed more light on the crucial role of risk sharing than their partial equilibrium counterparts. They generate different patterns of behavior by assigning speculators, hedgers, producers, etc. entirely different objective functions and different access to information. Speculation, hedging, industry choice and futures trading all arise endogenously in this model, whereas they are artificially imposed in the existing literature.

The modern thought on backwardation and contango[2] dates back to J. M. Keynes (1930, p.144), who wrote: "The quoted forward price,..., must fall below the anticipated future spot price by at least the amount of the normal backwardation." He considered backwardation an equivalent of a risk premium. Assuming that most hedgers were taking short positions, he concluded that the futures price has to be below the expected spot price in order to attract a sufficient number of long speculators to clear the market.

Formal models have been developed in an attempt to confirm or refute Keynes's conjecture. Two previous general equilibrium models have shown, as this paper does, that either backwardation or contango may occur. The previous models, however, are rather cumbersome with a very complicated structure. Danthine's (1978) seminal paper also uses agents who are exogenously destined to be either speculators or hedgers. Danthine shows that the bias is generated when speculators are endowed with more information than hedgers. The differential is a source of the speculators' expected profit. In our model, there is no exogenous industry formation or exogenous division of population. On the contrary, speculation arises endogenously.

Anderson and Danthine (1983) showed conditions that may lead to backwardation and contango. Results they obtained reflect a complicated exogenously-determined market structure. The Anderson and Danthine model is populated by four types of agents: speculators, producers, processors and storage companies. Each type has a different objective function. Uncertainty enters their model via two exogenous production and demand shocks. In contrast, the model presented in this paper generates both kinds of bias in an economy with agents who have identical objective functions, except for their degree of risk aversion. Due to the general equilibrium nature of our model, agents are exposed to the price risk and income risk generated by a single source of uncertainty, a production shock in one of the two industries.

Conventionally, commodity producers are expected to hedge by selling their output at a predetermined price. If they do not hedge or if they speculate by taking long positions, it is taken as evidence that they are risk loving, or they have access to private information.

[1] Optimal hedging and speculation in the presence of joint income and output risk have been studied by Grant (1985). Other related papers include Briys and Schlesinger (1993) who use state dependent preferences, Karp (1987) who studies dynamic hedging with stochastic production, Lapan, Moschini and Hanson (1991) consider simultaneous presence of futures and options markets. Haruna (1996) studies relationship between spot and futures prices in a simple long-run competitive industry partial equilibrium context with no exogenous speculators.

[2] Backwardation (contango) occurs when the current cash price is greater than (is less than) the futures price. In this model, backwardation (contango) refers to the relationship between the futures price and the expected spot price.

Results of this paper contradict conventional wisdom in two respects. First, what appears to be speculation may in fact be the behavior of a very risk averse individual. In our model, with all agents having the same information, a surprising and seemingly counter-intuitive pattern of futures trading arises as a result of combined exposure to price and income risk. The oversimplified conventional view of hedging and speculation has been pointed out by Fouda at al. (1999), who showed that different producers may take either short or long futures positions because they are exposed to a differential cash flow risk. In their model, producers differ from each other in the technology they use to produce the commodity. Risk aversion, however, plays no role in their results. In this paper, the risk exposure of all commodity producers is identical; they choose different futures positions because of their differential risk aversion.

Second, in general, there are agents who are worse off in terms of their expected utility when the futures market is available. The possibility of risk sharing in the futures market attracts entry of agents who would otherwise find the industry with stochastic output too risky. That leads to a greater output and a lower relative price of that good. As a result, other producers of that good who choose not to trade when futures trading is available must be worse off, because they sell their product at a lower spot price.

The existing theory of futures markets pays almost no attention to their welfare effects. The existence of futures markets is usually treated as ex-ante beneficial to all agents. Although real-world commodity producers have sometimes complained that futures markets harm them, the only rationale for this in existing models is ex-post price regret. In partial equilibrium, welfare does not seem to be an interesting issue because every producer can choose to stay away from the futures market and be no worse off than he would be in the absence of it. This view, however, neglects the effect futures markets may have on the long-run industry formation and subsequently on relative prices. If industry formation changes, the resulting change in relative prices will impact the welfare of virtually everyone in the economy. In this model, equivalent variation is used to study the effect of the futures market on the welfare of individual agents. We show that there may exist an identifiable group of endogenous speculators, and that when such a group arises, they benefit most.

The rest of the paper is organized in three sections. Equilibrium properties of the model with the aggregate production shock are shown in Section 2. The effect of the idiosyncratic production shock is analyzed in Section 3. The welfare effect of the futures market is investigated in Section 4.

2 Model With An Aggregate Production Shock

In this section, we study properties of the economy in which agents make all their production and consumption decisions in one period. Strong closed-form results are the benefit of the simplicity of the model.

Population

The economy is populated by a continuum of agents who have identical preferences except for their coefficient of absolute risk aversion. The agent's risk aversion is determined by his type, $a \in (\mu_a - \lambda_a, \mu_a + \lambda_a)$, where $0 < \mu_a - \lambda_a < \mu_a + \lambda_a < \infty$. Agents are uniformly distributed on the interval $(\mu_a - \lambda_a, \mu_a + \lambda_a)$; μ_a is the average degree of risk aversion and λ_a is the dispersion of risk aversion. The size of the population is normalized to 1.

Preferences

There are two goods, x and y. The utility function of an agent of type a is[3]

$$u = -\exp\left[-a c_x^{\beta} c_y^{1-\beta}\right],$$

where c_x is the consumption of good x and c_y is the consumption of good y.

Parameter β is the same for all agents. It determines the proportion of income agents spend on good x. Good x is a numeraire, its price is normalized to 1. Demand functions for goods x and y are $c_x = \beta I$, $c_y = \frac{(1-\beta)I}{p}$, where I is the agent's income. The relative price of good y, p, is expressed in units of good x. Substituting demand functions into the utility function gives the indirect utility function

$$V = \exp\left[-a \frac{\widehat{\beta}}{p^{1-\beta}} I\right],$$

where $\widehat{\beta} \equiv \beta^{\beta}(1-\beta)^{1-\beta}$.

Production And Income

Each agent possesses an indivisible unit of labor and supplies it inelastically. The agent must specialize in the production of either x or y. In the x-industry, the unit of labor is transformed into 1 unit of good x (a harmless normalization). If the agent devotes his unit of labor to the production of y, output, $\widetilde{Y} = \mu + \tilde{\lambda}$, is a random variable. For the sake of simplicity, it is assumed that $\widetilde{Y}$ has a Bernoulli distribution with the high state (good harvest) $\widetilde{Y} = \mu + \lambda$ occurring with probability 1/2 and the low state (bad harvest) $\widetilde{Y} = \mu - \lambda$ occurring with probability 1/2, where $\mu - \lambda > 0$.[4] The random shock is an aggregate shock; all of the y-producers have a good harvest or all of them have a bad one. Each agent sells the output at the market price and generates income $I_x = 1$, or $I_y = \widetilde{p}\widetilde{Y}$, respectively.

2.1 Industry Formation Without A Futures Market

Rational Expectations

Each agent chooses to enter one of the two industries. Then he produces in the industry of his choice, observes the realization of the random shock, sells his output in the spot market and consumes his preferred consumption bundle.

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[3] The validity of this approach to represent risk aversion of agents consuming many commodities has been established in Kihlstrom and Mirman (1974).

[4] Since a more general distribution of the two production states with probabilities q and $(1-q)$ does not affect the qualitative nature of results of this model, $q = 1/2$ is used for simplicity throughout this dissertation.

At the time agents choose the industry they will enter, they do not know the realization of $\widetilde{Y}$ or $\widetilde{p}$. Agents therefore choose the industry which promises greater expected utility EV:

$$EV^x = -\frac{1}{2}\exp\left[-a\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\right] - \frac{1}{2}\exp\left[-a\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\right], \quad (1.a)$$

$$EV^y = -\frac{1}{2}\exp\left[-a\widehat{\beta}\underline{p}^{\beta}(\mu+\lambda)\right] - \frac{1}{2}\exp\left[-a\widehat{\beta}\overline{p}^{\beta}(\mu-\lambda)\right] \quad (1.b)$$

The low price $\underline{p}$ corresponds to the high output state and the high price $\overline{p}$ to the low output state of the world. Agents' expectations are rational: the resulting distribution of relative price is as they expected.

Market Equilibrium

Let A be the measure of the subset of agents who produce x; $(1-A)$ is a measure of the subset of agents who produce y. Quantity demanded of good x is the sum of quantities demanded by x-producers and y-producers: $\beta A + \beta(1-A)\widetilde{p}\widetilde{Y}$. The supply of good x is equal to A. The market for good x clears when $\beta A + \beta(1-A)\widetilde{p}\widetilde{Y} = A$. This reduces to $\widetilde{p} = \frac{(1-\beta)}{\beta}\frac{A}{(1-A)}\frac{1}{\widetilde{Y}}$. When the market for x clears, the market for good y must clear by Walras law. The distribution of relative price $\widetilde{p}$ of good y is then

$$\widetilde{p} = \begin{cases} \underline{p} = \frac{(1-\beta)}{\beta}\frac{A}{(1-A)}\frac{1}{(\mu+\lambda)}, \text{with probability } \frac{1}{2} \\ \overline{p} = \frac{(1-\beta)}{\beta}\frac{A}{(1-A)}\frac{1}{(\mu-\lambda)}, \text{with probability } \frac{1}{2} \end{cases}. \quad (2)$$

Substituting expressions (2) into the utility function EV^y in (1.b) gives

$$EV^y = -\frac{1}{2}\exp\left[-a\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\frac{(1-\beta)}{\beta}\frac{A}{(1-A)}\right] - \frac{1}{2}\exp\left[-a\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\frac{(1-\beta)}{\beta}\frac{A}{(1-A)}\right]. \quad (1.c)$$

Definition 1 *The population is divided into a subset G of x-producers and a subset H of y-producers. An equilibrium is a set $(\{G,H\},\widetilde{p})$, where $\{G,H\}$ is a partition of the population such that $EV^x \geq EV^y$ holds for all $a \in G$, and $EV^x \leq EV^y$ holds for all $a \in H$. Relative price $\widetilde{p}$ satisfies (2), where the proportion of x-producers $A = \int_G d\nu(a)$ and the proportion of y-producers $(1-A) = \int_H d\nu(a)$. Measure ν is a uniform measure on $(\mu_a - \lambda_a, \mu_a + \lambda_a)$.*

Proposition 2 *An equilibrium partition $\{G,H\}$ is any partition that satisfies $A = \beta$. The equilibrium price in the two states of the world is $\overline{p} = \frac{1}{(\mu-\lambda)}$ and $\underline{p} = \frac{1}{(\mu+\lambda)}$, respectively.*

Proof. Note that the two expected utility functions (1.a) and (1.c) are equal to each other (for all a) if and only if $1 = \frac{(1-\beta)}{\beta}\frac{A}{(1-A)}$. This implies that $A = \beta$. If $A < (>)\beta$, all agents would want to enter the x-industry (y-industry), which is not consistent with equilibrium. The corresponding equilibrium price in the two output states is obtained by substituting $A = \beta$ in (2). This completes the proof. ■

If $A = \beta$, all agents are indifferent between entering the two industries. Any partition of agents that results in β agents producing x and $(1-\beta)$ agents producing y is consistent with equilibrium. This strong result is due to the unitary price elasticity of demand that eliminates income uncertainty and due to the fact that the production shock is the same for all producers. Recall that the x-producer's income is $I^x = 1$. In equilibrium, the y-producer's income is $I^y = \underline{p}\,(\mu + \lambda) = 1$ when the output is high and $I^y = \overline{p}\,(\mu - \lambda) = 1$ when the output is low. The fact that it is the y-industry that has the random output does not make it a "riskier" industry. As producers, all agents have the same income. As consumers, they face the same price uncertainty.

Note that the equilibrium proportion of x-producers and y-producers in the population is determined by the split of agents' income. Portion βI is used to buy x and portion $(1-\beta)\,I$ is used to buy y. It is not affected by the presence of the aggregate shock; it would be the same if output were deterministic in both industries .

2.2 Industry Formation With A Futures Market

Suppose that, at the time he chooses the industry, each agent has the opportunity to trade futures on commodity y. Since the only source of risk here is the aggregate shock resulting in two states of the world, the futures contract is equivalent to a contingent claims contract.

Let z be the number of units of y an agent contracts to buy and let p_F be the futures price. Agents who choose $z > 0$ take long positions (they will buy z units of y at the price p_F), while those who choose $z < 0$ take short positions (they will sell $|z|$ units of y at the price p_F).[5] The delivery will take place after realizations of the random output $\widetilde{Y}$ and random price $\widetilde{p}$ have been observed. The income of an agent is then given by

$$
\begin{aligned}
I^x &= 1 + z(\widetilde{p} - p_F), \text{ or} && (3.a)\\
I^y &= pY + z(\widetilde{p} - p_F), && (3.b)
\end{aligned}
$$

respectively.

Choice Of The Futures Position z

Let $\overline{D} \equiv \overline{p} - p_F$ and $\underline{D} \equiv p_F - \underline{p}$. The expected utility function of an agent who enters the x-industry is

$$EV^x = \underset{z}{MAX}\left\{-\frac{1}{2}\exp\left[-a\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\left(1 - z\underline{D}\right)\right] - \frac{1}{2}\exp\left[-a\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\left(1 + z\overline{D}\right)\right]\right\}, \qquad (4.a)$$

while the y-producer's utility function is

$$EV^y = \underset{z}{MAX}\left\{\begin{array}{c} -\frac{1}{2}\exp\left[-a\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\left(\underline{p}\,(\mu+\lambda) - z\underline{D}\right)\right] \\ -\frac{1}{2}\exp\left[-a\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\left(\overline{p}\,(\mu-\lambda) + z\overline{D}\right)\right] \end{array}\right\}. \qquad (4.b)$$

[5] The contracts are referred to as futures even though the setup is too general to distinguish them from forward contracts.

Both functions are concave in z. Agents choose their optimal futures positions

$$z^x = \arg\max EV^x \text{ and } z^y = \arg\max EV^y$$

subject to the constraint $I^x, I^y \geq 0$. Any potential loss resulting from a futures position must be backed by income from production because agents are not allowed to default on their obligation in any state of the world. When the solutions are interior, first-order conditions give

$$z^x = \frac{\left(\frac{\widehat{\beta}}{\underline{p}^{1-\beta}} - \frac{\widehat{\beta}}{\bar{p}^{1-\beta}}\right) - \frac{1}{a}\ln K}{\frac{\widehat{\beta}}{\bar{p}^{1-\beta}}\overline{D} + \frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\underline{D}}, \text{ and} \tag{5.a}$$

$$z^y = \frac{\left(\widehat{\beta}\underline{p}^{\beta}(\mu+\lambda) - \widehat{\beta}\bar{p}^{\beta}(\mu-\lambda)\right) - \frac{1}{a}\ln K}{\frac{\widehat{\beta}}{\bar{p}^{1-\beta}}\overline{D} + \frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\underline{D}}, \tag{5.b}$$

where $K \equiv \frac{\underline{D}}{\overline{D}}\left(\frac{\bar{p}}{\underline{p}}\right)^{1-\beta} \equiv \frac{(p_F - \underline{p})}{(\bar{p} - p_F)}\left(\frac{\bar{p}}{\underline{p}}\right)^{1-\beta}$. Substituting optimal futures positions (5.a) and (5.b) into the functions (4.a) and (4.b) gives

$$EV^x = -2MK^{1+M}\exp\left[-a\frac{\widehat{\beta}\left(\bar{p} - \underline{p}\right)}{\overline{D}\underline{p}^{1-\beta} + \underline{D}\bar{p}^{1-\beta}}\right], \text{ and} \tag{6.a}$$

$$EV^y = -2MK^{1+M}\left(\exp\left[-a\frac{\widehat{\beta}\left(\bar{p} - \underline{p}\right)}{\overline{D}\underline{p}^{1-\beta} + \underline{D}\bar{p}^{1-\beta}}\right]\right)^{\left(\frac{(1-\beta)}{\beta}\frac{A}{(1-A)}\right)}, \tag{6.b}$$

where $M \equiv \frac{\frac{\overline{D}}{\bar{p}^{1-\beta}}}{\frac{\overline{D}}{\bar{p}^{1-\beta}} + \frac{\underline{D}}{\underline{p}^{1-\beta}}}$.

Notice that EV^y differs from EV^x only by the exponent of $\frac{(1-\beta)}{\beta}\frac{A}{(1-A)}$. If the exponent is greater (smaller) than one, all agents want to enter the y-industry (x-industry), which is not consistent with the equilibrium. All agents are indifferent between the two industries if and only if $\frac{(1-\beta)}{\beta}\frac{A}{(1-A)} = 1$.

Equilibrium in the Futures Market

The futures market clears when every long futures position is offset by a short futures position. The sum of all positions must therefore add up to zero. If the sum of the futures positions held by the x-producers is $\int_G z^x d\nu(a)$ and the sum of the futures positions held by the y-producers is $\int_H z^y d\nu(a)$, then the futures price p_F must solve

$$\int_G z^x d\nu(a) + \int_H z^y d\nu(a) = 0. \tag{7}$$

Equilibrium in the Spot Market

Market demand for good x consists of the quantity

$$\beta \int_G \left(1 + (\widetilde{p} - p_F) z^x\right) d\nu(a)$$

demanded by x-producers and the quantity

$$\beta \int_H \left(\widetilde{p}\widetilde{Y} + (\widetilde{p} - p_F) z^y\right) d\nu(a)$$

demanded by y-producers. The supply of good x is A. Quantity demanded of good x equals quantity supplied when $\widetilde{p}$ solves

$$\beta \int_G \left(1 + (\widetilde{p} - p_F) z^x\right) d\nu(a) + \beta \int_H \left(pY + (\widetilde{p} - p_F) z^y\right) d\nu(a) = A.$$

This can be written as

$$\beta A + \beta (1 - A)\, pY + \beta(\widetilde{p} - p_F) \left[\int_G z^x d\nu(a) + \int_H z^y d\nu(a)\right] = A.$$

When the futures market clears, (7) holds and the spot market clearing condition reduces to

$$\beta A + \beta (1 - A)\, pY = A.$$

The equilibrium relative price $\widetilde{p}$ then satisfies Eq. (2):

$$\widetilde{p} = \begin{cases} \underline{p} = \frac{(1-\beta)}{\beta} \frac{A}{(1-A)} \frac{1}{(\mu+\lambda)}, \text{with probability } \frac{1}{2} \\ \overline{p} = \frac{(1-\beta)}{\beta} \frac{A}{(1-A)} \frac{1}{(\mu-\lambda)}, \text{with probability } \frac{1}{2} \end{cases}.$$

Definition 3 *Equilibrium is a set $(\{G, H\}, \widetilde{p}, p_F)$, where $\{G, H\}$ is a partition of the population such that $EV^x \geq EV^y$ holds for all $a \in G$, and $EV^x \leq EV^y$ holds for all $a \in H$. Relative price $\widetilde{p}$ satisfies Eq. (2) and futures price p_F satisfies Eq. (7); both the spot market and the futures market clear.*

When the no-default constraint is not binding, equilibrium is described in the following proposition.

Proposition 4 ***(A)*** *The equilibrium partition is any $\{G, H\}$ such that $A = \beta$. Equilibrium prices in the two states of the world are $\overline{p} = \frac{1}{(\mu-\lambda)}$ and $\underline{p} = \frac{1}{(\mu+\lambda)}$, respectively.*

(B) *Each agent's choice of futures position is independent of the industry he enters. There is a unique agent $\hat{a} = \frac{2\lambda_a}{\ln(\mu_a+\lambda_a)-\ln(\mu_a-\lambda_a)}$ who chooses $z = 0$ (see Figure 1). All $a < (>)\,\hat{a}$ choose $z < (>)0$; more risk-averse agents take long positions and less risk-averse agents take short positions.*

(C) *The equilibrium futures price is*

$$p_F = \left\{ \frac{1}{1 + \left(\frac{\mu+\lambda}{\mu-\lambda}\right)^{1-\beta} \left(\frac{\exp\left[-\frac{2\lambda_a}{ln(\mu_a+\lambda_a)-\ln(\mu_a-\lambda_a)}\widehat{\beta}(\mu+\lambda)^{1-\beta}\right]}{\exp\left[-\frac{2\lambda_a}{ln(\mu_a+\lambda_a)-\ln(\mu_a-\lambda_a)}\widehat{\beta}(\mu-\lambda)^{1-\beta}\right]} \right)} \right\} \frac{1}{(\mu-\lambda)}$$
$$+ \left\{ \frac{\left(\frac{\mu+\lambda}{\mu-\lambda}\right)^{1-\beta} \left(\frac{\exp\left[-\frac{2\lambda_a}{ln(\mu_a+\lambda_a)-\ln(\mu_a-\lambda_a)}\widehat{\beta}(\mu+\lambda)^{1-\beta}\right]}{\exp\left[-\frac{2\lambda_a}{ln(\mu_a+\lambda_a)-\ln(\mu_a-\lambda_a)}\widehat{\beta}(\mu-\lambda)^{1-\beta}\right]} \right)}{1 + \left(\frac{\mu+\lambda}{\mu-\lambda}\right)^{1-\beta} \left(\frac{\exp\left[-\frac{2\lambda_a}{ln(\mu_a+\lambda_a)-\ln(\mu_a-\lambda_a)}\widehat{\beta}(\mu+\lambda)^{1-\beta}\right]}{\exp\left[-\frac{2\lambda_a}{ln(\mu_a+\lambda_a)-\ln(\mu_a-\lambda_a)}\widehat{\beta}(\mu-\lambda)^{1-\beta}\right]} \right)} \right\} \frac{1}{(\mu+\lambda)}.$$

(D) *The equilibrium futures price p_F is an increasing function of the average coefficient of risk aversion μ_a and of the aggregate shock λ. It is a decreasing function of the mean output μ.*

(E) *The futures price is equal to the expected spot price if and only if*

$$\frac{2\lambda_a}{\ln(\mu_a+\lambda_a)-\ln(\mu_a-\lambda_a)} = \frac{\ln(\mu+\lambda)^{1-\beta} - \ln(\mu-\lambda)^{1-\beta}}{\widehat{\beta}(\mu+\lambda)^{1-\beta} - \widehat{\beta}(\mu-\lambda)^{1-\beta}}.$$

If the left-hand side is smaller (greater) than the right-hand side, then $p_F > (<)\,E(p)$.

Proof. See Appendix A. ■

Figure 1 depicts each agent's futures position as a function of his coefficient of absolute risk aversion, which is on the horizontal axis. Increasing values of a represent greater risk aversion. Positive futures positions are long, negative positions are short. The area under the curve represents the sum of all futures positions taken by agents of different types. In equilibrium, the area enclosed by the long positions has to be equal in size to the area enclosed by the short positions.

The intuition behind the equilibrium in the product market is the same as in the previous section. The result is again driven by the unitary price elasticity of demand and the aggregate nature of the production shock. Agents in both industries face the same price risk. The income from production, measured in units of good x, is certain and is the same in both industries.

The futures market gives more risk averse agents the opportunity to insure against the worst-case scenario, which is the state of the world where the output of y is low and

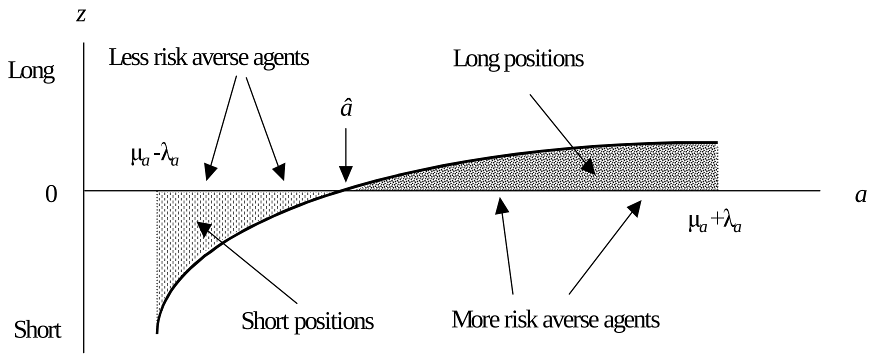


Figure 1: Futures positions held by agents

its price is high. In this case, it is better to buy y at the price $p_F < \bar{p}$. For a risk averse agent, the additional income from futures trading in this bad state compensates for the high price of y. That is why more risk-averse agents take long positions. The less risk-averse agents, on the other hand, take the opportunity to increase their expected utility by taking short positions and exposing themselves to a greater risk in utility.

Properties Of The Equilibrium Futures Price p_F

The monotonicity of p_F in μ, λ, μ_a and λ_a is intuitive. First, the relative price of y decreases with the mean output μ in every state of the world. That is why the futures price decreases as well.

The increase in aggregate shock λ increases the variability of the relative price of y. All agents demand a larger z and the futures price must increase to clear the futures market.

A larger μ_a represents a more risk-averse population, that chooses greater long positions and smaller short positions. The equilibrium futures price must therefore increase to clear the market.

The increase in the dispersion of risk aversion λ_a increases both the number of short positions less risk averse-agents demand and the number of long positions more risk-averse agents demand. Because of concavity of $z(a)$, larger λ_a results in excess short positions. To clear the futures market, p_F must decrease.

Futures Price and Expected Spot Price

As Proposition 4 shows, futures price is not an unbiased estimator of the expected spot price, $E(p) = \frac{1}{2}\bar{p} + \frac{1}{2}\underline{p}$. The presence of backwardation or contango, defined here as the relation between the futures price and the expected spot price, can be tied directly to the distribution of the random output of y, and to the risk aversion of the population.[6]

[6] Most of the current literature studies backwardation as an increasing trend and contango as a decreasing trend in the term structure of futures prices (e.g. Kolb, 1992). Originaly, Keynes and Hicks considered normal backwardation to be the equivalent of a risk premium. Keynes (1930, p.144) wrote: " The quoted forward price,..., must fall below the anticipated future spot price by at least the amount of the normal backwardation."

The effect of the average coefficient of risk aversion, μ_a, on the futures price is intuitive. Equilibrium futures price is a weighted average of $\overline{p}$ and $\underline{p}$. If the population consists of strongly risk averse individuals (μ_a is relatively large) who tend to reduce their risk in utility by taking larger long positions, the futures market clears at a futures price that is relatively high, close to $\overline{p}$. This results in contango, the upward bias of the futures price $p_F > E(p)$. On the other hand, if the population consists of less risk averse agents (μ_a is relatively small) who tend to increase their risk in utility by taking large short positions, the equilibrium is achieved at a relatively low futures price, close to $\underline{p}$. The futures price is biased downward and backwardation $p_F < E(p)$ is observed.

The relationship between the futures price and the expected spot price is represented in the following three diagrams. They show the ratio $p_F/E(p)$ as a function of the aggregate shock for three different values of the average coefficient of risk aversion. When backwardation occurs, $p_F/E(p) < 1$, while $p_F/E(p) > 1$ shows contango.

The values $\mu = 1$ and $\lambda_a = 1$ are the same in all three diagrams. In Figure 2, average risk aversion, $\mu_a = 2$, is weak; in Figure 3, $\mu_a = 3$ is moderate; Figure 4 shows strong average risk aversion, $\mu_a = 4$.

Equilibrium With A Binding No-Default Constraint

Even absent the no-default constraint, each agent's choice of the futures position is bounded from above by

$$z = \frac{\left(\widehat{\beta}(\mu+\lambda)^{1-\beta} - \widehat{\beta}(\mu-\lambda)^{1-\beta}\right)}{\widehat{\beta}(\mu-\lambda)^{1-\beta}\,\overline{D} + \widehat{\beta}(\mu+\lambda)^{1-\beta}\,\underline{D}},$$

for all $a \in (\mu_a - \lambda_a, \mu_a + \lambda_a)$. The expression on the right-hand side is a position an extremely risk averse agent ($a \to \infty$) would choose; he would be fully insured, purchasing all of his y in the futures market, none in the spot market. Therefore, the no default constraint never binds for those who take long positions.

There is, however, no corresponding lower bound on the short futures position an agent with low risk aversion might choose. The income of an agent in the low output/high price of y state of the world is

$$I^x = I^y = 1 + \overline{D}z.$$

A short position must be constrained by $z \geq -\frac{1}{\overline{D}}$. An agent with a low risk aversion (a sufficiently small a), would choose $z < -\frac{1}{\overline{D}}$, making his income negative in the low-output state. When the constraint is binding for the least risk averse agents, properties of the equilibrium described in parts A) and B) of Proposition 4 do not change. The futures price, however, must be greater than the value given by the formula in part C) of Proposition 4.

Next section shows that the properties of equilibrium are different when the output of good y is subject to a combination of the aggregate and idiosyncratic shocks.

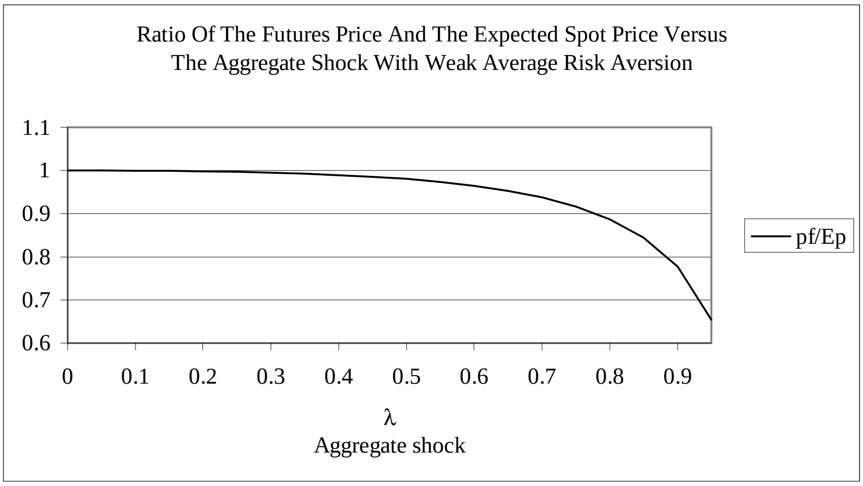


Figure 2: Weak average risk aversion ($\mu_a = 2$) results in backwardation for all λ

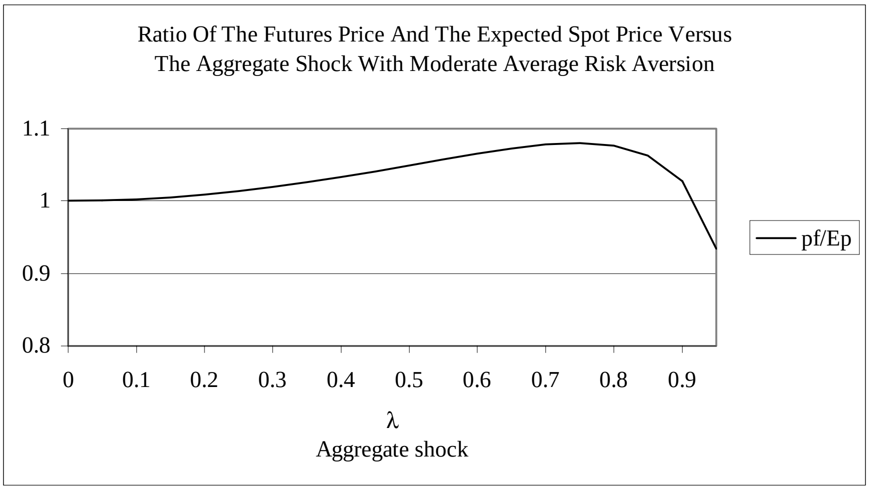


Figure 3: Moderate average risk aversion ($\mu_a = 3$) results in backwardation for large λ, contango for small λ

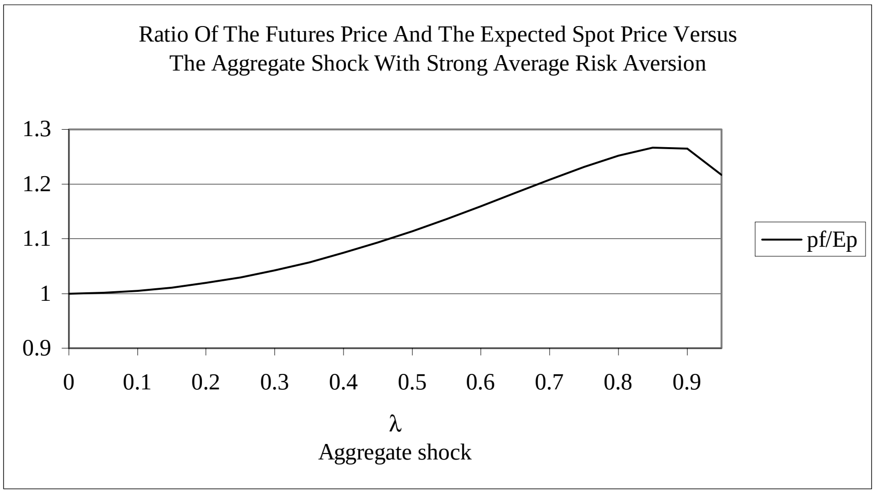


Figure 4: Strong average risk aversion ($\mu_a = 4$) results in contango for all λ

3 Model With Aggregate And Idiosyncratic Shocks

Commodity producers are often exposed to output risk that is not related to the overall market conditions. For example, even when the country enjoys a bumper crop, an individual farmer may loose his crop to a localized flood. In this section, the effects of such idiosyncratic risk on agents' behavior and market equilibrium are examined.

3.1 Industry Formation Without A Futures Market

This section considers the effects of a combination of aggregate and idiosyncratic shocks on equilibrium, when there is no futures market. The idiosyncratic shock is modeled as a Bernoulli random variable $\tilde{\omega}_a$ that has realizations $\omega > 0$ with probability $\frac{1}{2}$ and $-\omega$ with probability $\frac{1}{2}$. While all y-producers experience the same realization of the aggregate shock, the realization of the idiosyncratic shock differs among y-producers. It is assumed that the sum of realizations of the idiosyncratic shock over the subset H of all y-producers is zero: $\int_H \widetilde{\omega}_a d\nu(a) = 0$. There are now four possible realizations of a y-producer's random output.

$$\widetilde{Y} = \begin{cases} \mu + \lambda + \omega, \text{ with probability } 1/4 \\ \mu + \lambda - \omega, \text{ with probability } 1/4 \\ \mu - \lambda + \omega, \text{ with probability } 1/4 \\ \mu - \lambda - \omega, \text{ with probability } 1/4 \end{cases}.$$

It is assumed that $0 < \omega \leq \mu - \lambda$ for all y-producers; output is non-negative in the worst state. The expected utility function of an x-producer is identical to (1):

$$EV^x = -\frac{1}{2}\exp\left[-a\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\right] - \frac{1}{2}\exp\left[-a\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\right].$$

The y-producer's expected utility function is

$$\begin{aligned} EV^y &= \frac{1}{2}\left(\frac{1}{2}\exp\left[-a\widehat{\beta}\underline{p}^{\beta}\left(\mu+\lambda+\omega\right)\right] + \frac{1}{2}\exp\left[-a\widehat{\beta}\underline{p}^{\beta}\left(\mu+\lambda-\omega\right)\right]\right) \\ &\quad + \frac{1}{2}\left(\frac{1}{2}\exp\left[-a\widehat{\beta}\overline{p}^{\beta}\left(\mu-\lambda+\omega\right)\right] + \frac{1}{2}\exp\left[-a\widehat{\beta}\overline{p}^{\beta}\left(\mu-\lambda-\omega\right)\right]\right). \end{aligned}$$

Expressions containing idiosyncratic shock ω can be factored out as hyperbolic cosine and EV^y can be conveniently written as

$$EV^y = -\frac{1}{2}\underline{C}\exp\left[-a\widehat{\beta}\underline{p}^{\beta}\left(\mu+\lambda\right)\right] - \frac{1}{2}\overline{C}\exp\left[-a\widehat{\beta}\overline{p}^{\beta}\left(\mu-\lambda\right)\right], \tag{8}$$

where

$$\begin{aligned} \underline{C} &\equiv \cosh\left(a\widehat{\beta}\underline{p}^{\beta}\omega\right) = \frac{\exp\left[a\widehat{\beta}\underline{p}^{\beta}\omega\right] + \exp\left[-a\widehat{\beta}\underline{p}^{\beta}\omega\right]}{2}, \text{ and} \\ \overline{C} &\equiv \cosh\left(a\widehat{\beta}\overline{p}^{\beta}\omega\right) = \frac{\exp\left[a\widehat{\beta}\overline{p}^{\beta}\omega\right] + \exp\left[-a\widehat{\beta}\overline{p}^{\beta}\omega\right]}{2}. \end{aligned}$$

Market Equilibrium

Quantity demanded of good x is the sum of the quantities demanded by x-producers and y-producers:

$$\beta A + \beta\int_H \widetilde{p}\left(\mu+\widetilde{\lambda}+\widetilde{\omega}_a\right)d\nu(a).$$

Since $\int_H \widetilde{\omega}_a d\nu\left(a\right) = 0$, quantity demanded reduces to

$$\beta A + \beta\int_H \widetilde{p}\left(\mu+\widetilde{\lambda}\right)d\nu(a).$$

The supply of good x is equal to A. The market for good x clears when

$$\beta A + \beta\left(1-A\right)\underline{p}\left(\mu+\lambda\right) = A$$

if the aggregate shock is high or

$$\beta A + \beta\left(1-A\right)\overline{p}\left(\mu-\lambda\right) = A$$

if the aggregate shock is low. The distribution of the relative price of good y is then given by Eq. (2):

$$\widetilde{p} = \begin{cases} \underline{p} = \frac{(1-\beta)}{\beta} \frac{A}{(1-A)} \frac{1}{(\mu+\lambda)}, \text{with probability } \frac{1}{2} \\ \bar{p} = \frac{(1-\beta)}{\beta} \frac{A}{(1-A)} \frac{1}{(\mu-\lambda)}, \text{with probability } \frac{1}{2} \end{cases}.$$

The market for good y must clear by Walras law.

Proposition 5 ***A)*** *In equilibrium, $A > \beta$. There is exactly one agent*

$$a' \in (\mu_a - \lambda_a, \mu_a + \lambda_a)$$

who is indifferent between the two industries. Less risk averse agents $a < a'$ enter the y-industry and more risk averse agents $a > a'$ enter the x-industry.

B) *The equilibrium proportion A is an increasing function of the idiosyncratic shock ω and the average coefficient of risk aversion μ_a. It is a decreasing function of the expected output μ. If $A > (<) 1/2$, it is a decreasing (increasing) function of dispersion of risk aversion λ_a.*

Proof. See Appendix B. ■

The analysis in Section 2 showed that, in the absence of the idiosyncratic shock, equilibrium industry formation was determined exclusively by the proportion of income, β, agents spend on good x. Proposition 5 shows that the idiosyncratic shock changes properties of the equilibrium industry formation substantially. First, the proportion of x-producers increases and the proportion of y-producers decreases compared to the situation with no idiosyncratic shock. Since this reduces the supply of good y and increases its relative price, the y-producer's expected income is greater than that of an x-producer. This compensates y-producers for the added income risk they face due to the idiosyncratic shock. Furthermore, it is no longer the case that agents are indifferent between the two industries. More risk averse agents prefer less risky income in the x-industry, less risk-averse agents choose greater expected, albeit riskier, income in the y-industry.

The sensitivity of equilibrium industry formation to changes in parameter values is demonstrated in the following examples. In Figure 5 larger idiosyncratic shocks result in a greater proportion of x-producers A. The other parameter values, held constant in Figure 5, are $\mu = 1$, $\lambda = 0.2$, $(\mu_a - \lambda_a, \mu_a + \lambda_a) = (1, 4)$, and $\beta = 0.5$. As the magnitude of the idiosyncratic shock becomes potentially catastrophic to y-producers, the fraction of agents in the x-industry increases substantially.

The monotonicity of A in the expected output μ is also intuitive. When μ increases, other things constant, the impact of the idiosyncratic shock on the income of y-producers diminishes. Less risky income attracts more agents into the y-industry, thus reducing A.

Proposition 5 also states that the greater the average coefficient of risk aversion, the larger the proportion of x-producers. When a population consists of more risk averse individuals, the risk premium needed to attract agents to the riskier y-industry must be greater. That premium is provided by a higher relative price of y, resulting from a higher A and the reduced supply of y.

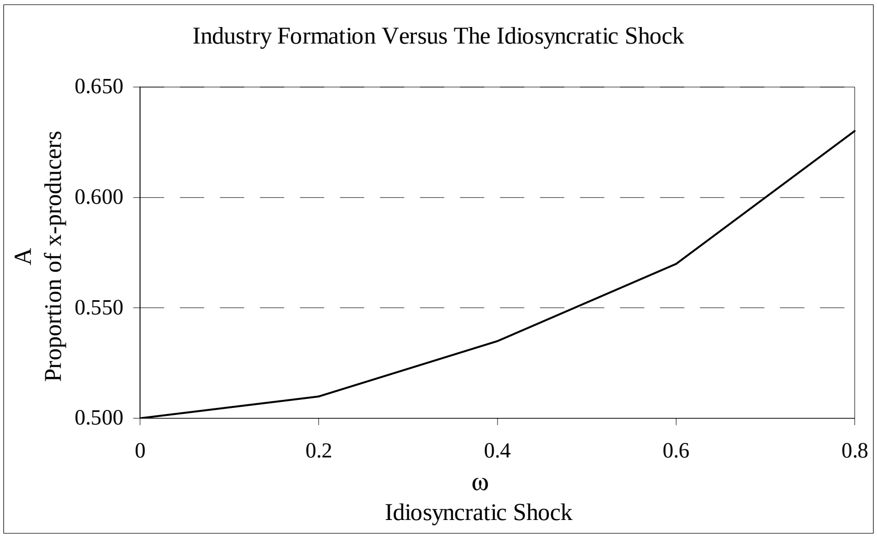


Figure 5: Proportion of x-producers increases with idiosyncratic shock

Figure 6 shows increasing equilibrium values of A corresponding to increasing values of the average coefficient of risk aversion. The dispersion of the coefficient of risk aversion is $\lambda_a = 1$. Production is again given by $\mu = 1$, $\lambda = 0.2$, and the idiosyncratic shock is fixed at $\omega = 0.7$. Agents spend proportion $\beta = 0.5$ on good x. The example demonstrates that a population with a high average coefficient of risk aversion needs a higher relative price of the commodity to attract agents into the y-industry.

The effect of the increasing dispersion of the coefficient of risk aversion λ_a on the industry formation depends on the value of A. To explain why, let's consider the marginal agent a' who is initially indifferent between the two industries. When $A > \frac{1}{2}$, $a' < \mu_a$. When λ_a increases, the agent who had been a' is displaced to a smaller degree of risk aversion. Ceteris paribus, he now wishes to enter the y-industry, reducing A. When $A < \frac{1}{2}$ and $a' > \mu_a$, agent a' is displaced to a greater degree of risk aversion and wishes to enter the x-industry, increasing A.

Figure 7 shows an example with initial $A > 1/2$: equilibrium values of A decrease as the dispersion of the coefficient of risk aversion λ_a increases. The average coefficient of risk aversion $\mu_a = 2$. Production is again given by $\mu = 1$, $\lambda = 0.2$ and $\omega = 0.7$. The proportion of income spent on good x is $\beta = 0.8$. The risk premium needed to attract agents into the y-industry becomes smaller, driving down the relative price of y and the proportion A.

Figure 8 shows an example with initial $A < 1/2$: equilibrium values of A increase as the dispersion of the coefficient of risk aversion λ_a increases. The proportion spent on good x is $\beta = 0.2$, all other parameters are the same. The risk premium needed to attract agents into the y-industry becomes larger, driving up the relative price of y and the proportion A.

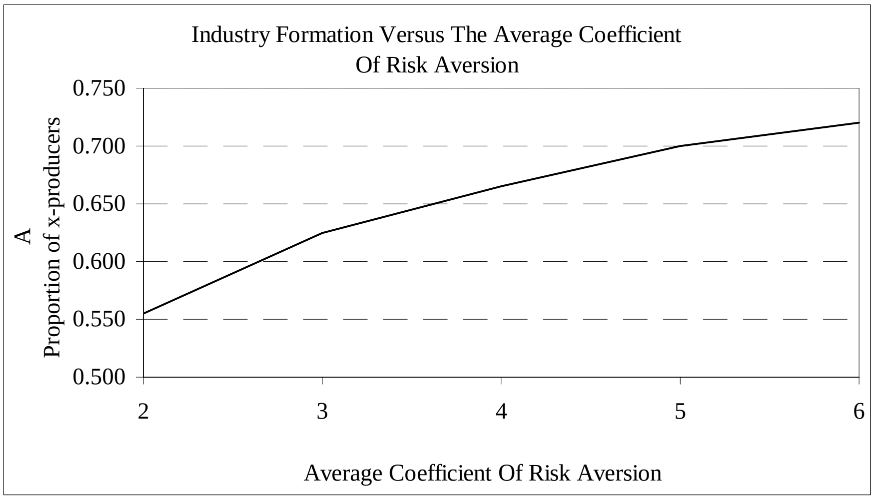


Figure 6: Proportion of x-producers increases with the average coefficient of absolute risk aversion

3.2 Industry Formation With A Futures Market

In Section 2 we showed that the introduction of a futures market does not change equilibrium industry formation in the absence of an idiosyncratic shock because agents were exposed to the same income risk in both industries.

In the presence of the idiosyncratic shock, however, y-producers are exposed to an extra element of income risk. This risk drives the most risk averse y-producers out of the industry and the supply of y decreases. The resulting higher relative price of y is the source of a premium y-producers earn for bearing the extra risk.

In this section, we will show that, in the presence of the idiosyncratic shock, having a futures market can affect industry formation.

Choice Of The Futures Position z

Each agent's utility maximization problem is analogous to that presented in Section 2. An agent who enters the x-industry chooses a futures position z in order to maximize the expected utility function

$$EV^x = -\frac{1}{2}\exp\left[-a\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\left(1 - z\underline{D}\right)\right] - \frac{1}{2}\exp\left[-a\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\left(1 + z\overline{D}\right)\right], \tag{9.a}$$

while a y-producer chooses z that maximizes

$$\begin{aligned} EV^y &= -\frac{1}{2}\underline{C}\exp\left[-a\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\left(\underline{p}\left(\mu+\lambda\right) - z\underline{D}\right)\right] \\ &\quad -\frac{1}{2}\overline{C}\exp\left[-a\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\left(\bar{p}\left(\mu-\lambda\right) + z\overline{D}\right)\right]. \end{aligned} \tag{9.b}$$

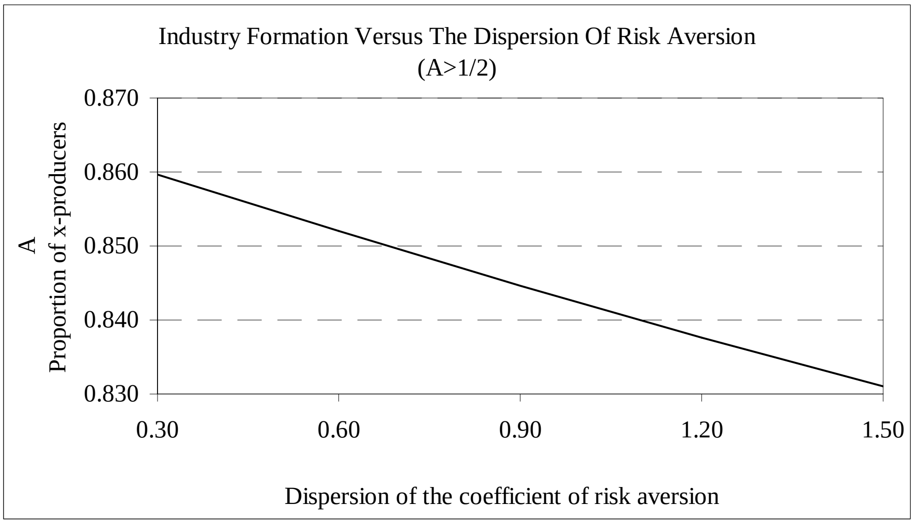


Figure 7: When $A > 1/2$, proportion of x-producers decreases with dispersion of coefficient of absolute risk aversion

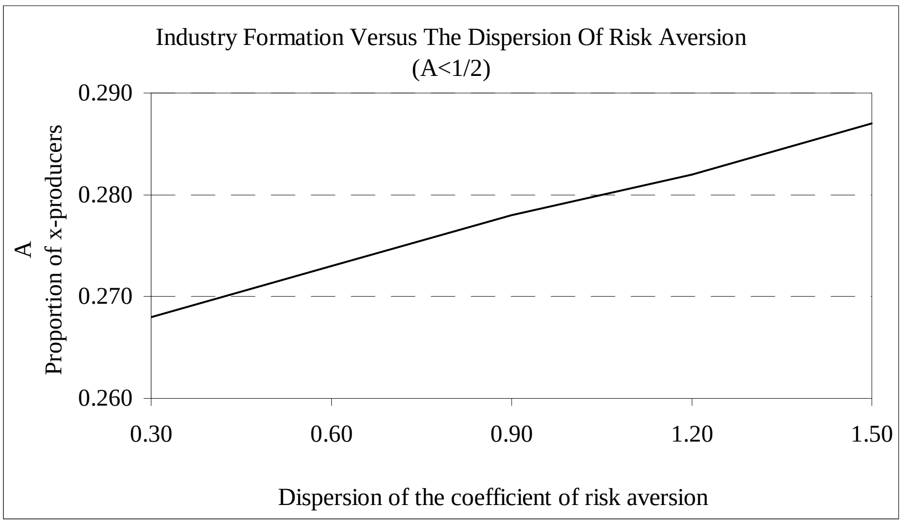


Figure 8: When $A < 1/2$, proportion of x-producers increases with dispersion of coefficient of absolute risk aversion

Recall that $\overline{D} \equiv \bar{p} - p_F$ and $\underline{D} \equiv p_F - \underline{p}$. When the solutions are interior, first-order conditions give

$$
\begin{aligned}
z^x &= \frac{\left(\frac{\widehat{\beta}}{\underline{p}^{1-\beta}} - \frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\right) - \frac{1}{a}\ln K^x}{\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\overline{D} + \frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\underline{D}}, \text{ and} \\
z^y &= \frac{\left(\widehat{\beta}\underline{p}^{\beta}\left(\mu+\lambda\right) - \widehat{\beta}\overline{p}^{\beta}\left(\mu-\lambda\right)\right) - \frac{1}{a}\ln K^y}{\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\overline{D} + \frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\underline{D}},
\end{aligned}
$$

where $K^x \equiv \frac{\overline{D}}{\underline{D}}\left(\frac{\bar{p}}{\underline{p}}\right)^{1-\beta}$ and $K^y \equiv \frac{\overline{C}}{\underline{C}}\frac{\overline{D}}{\underline{D}}\left(\frac{\bar{p}}{\underline{p}}\right)^{1-\beta}$.

Equilibrium in the Futures Market

As in Section 2, the futures market clears when the sum of all futures positions held by x-producers and y-producers is zero. The equilibrium futures price p_F solves Eq. (7):

$$\int_G z^x d\nu(a) + \int_H z^y d\nu(a) = 0.$$

Equilibrium in the Spot Market

The total demand for good x consists of the quantity

$$\beta \int_G \left(1 + (\widetilde{p} - p_F) z^x\right) d\nu(a)$$

demanded by x-producers and the quantity

$$\beta \int_H \left(pY + (\widetilde{p} - p_F) z^y\right) d\nu(a)$$

demanded by y-producers. The supply of good x is A. The quantity demanded of good x equals the quantity supplied when $\widetilde{p}$ solves

$$\beta \int_G \left(1 + (\widetilde{p} - p_F) z^x\right) d\nu(a) + \beta \int_H \left(\widetilde{p}\left(\mu + \widetilde{\lambda} + \widetilde{\omega}_a\right) + (\widetilde{p} - p_F) z^y\right) d\nu(a) = A.$$

This can be written as

$$\beta A + \beta\left(1 - A\right)\widetilde{p}\left(\mu + \widetilde{\lambda}\right) + \beta(\widetilde{p} - p_F)\left[\int_G z^x d\nu(a) + \int_H z^y d\nu(a)\right] = A.$$

When the futures market clears, Eq. (7) holds and the spot market clearing condition reduces to

$$\beta A+\beta\left(1-A\right)\widetilde{p}\left(\mu+\widetilde{\lambda}\right)=A.$$

The equilibrium relative price $\widetilde{p}$ then satisfies Eq. (2):

$$\widetilde{p}=\begin{cases} \underline{p}=\frac{(1-\beta)}{\beta}\frac{A}{(1-A)}\frac{1}{(\mu+\lambda)}, \text{with probability } \frac{1}{2} \\ \bar{p}=\frac{(1-\beta)}{\beta}\frac{A}{(1-A)}\frac{1}{(\mu-\lambda)}, \text{with probability } \frac{1}{2} \end{cases}.$$

When the no-default condition is not binding, properties of the equilibrium industry formation are described in the following proposition.

Proposition 6 *A) When the no-default condition is not binding, there is exactly one agent*

$$a'\in\left(\mu_a-\lambda_a,\mu_a+\lambda_a\right)$$

who is indifferent between the two industries. All agents $a<a'$ enter the y-industry and all agents $a>a'$ enter the x-industry.

B) In equilibrium, the proportion of x-producers $A>\beta$.

Proof. See Appendix C. ■

Figure 9 shows an example of the pattern of futures trading and industry formation in an economy populated by agents on the interval $(\mu_a-\lambda_a,\mu_a+\lambda_a)=(0.5,4)$. The coefficient of absolute risk aversion is on the horizontal axis, which is common for both diagrams. The futures position each agent takes is measured on the vertical axis in the top diagram. Positive values represent long positions and negative values are short positions. The shaded area in the bottom diagram depicts agents producing x, while the clear area depicts agents producing y. A small group of agents decides to switch their industry choice when they start trading futures. They are depicted as a thin dark bar dividing x-producers from the y-producers. Good y is produced with a technology that has a mean $\mu=1$, the aggregate shock $\lambda=0.2$ and the idiosyncratic shock $\omega=0.6$. Agents spend proportion $\beta=0.8$ on good x.

The more risk averse agents choose to enter the x-industry with non-random output, while less risk averse agents become y-producers. Among the x-producers, there are relatively more risk averse long hedgers, who lock in the price at which they buy y. Relatively less risk averse x-producers speculate by taking uncovered short positions – they lock in the price at which they sell good y they do not have.

On the other hand, y-producers show a surprising pattern of futures trading that deserves special attention. Conventionally, the commodity producers are expected to hedge (reduce their risk exposure) by taking short futures positions. For example, Bodie and Merton (1998, p. 224) wrote: "...farmers who sell their future crops before the harvest at a fixed price to eliminate the risk of a low price at a harvest time... are hedging their exposure to the price risk of their crops." Let's now compare the conventional wisdom with the pattern of futures positions y-producers take in Figure 9. The more

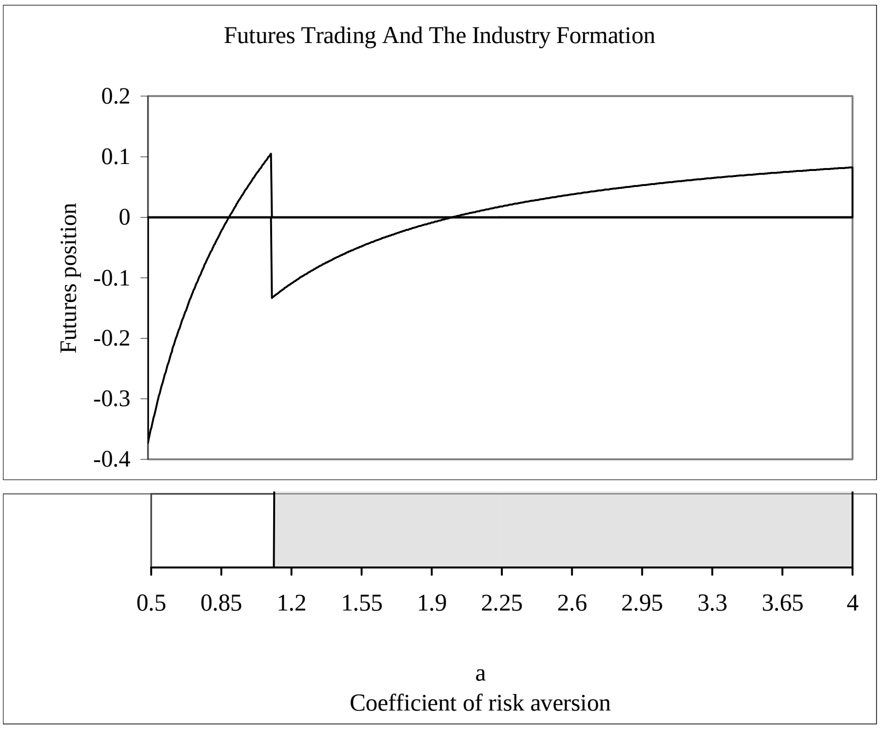


Figure 9: More risk averse agents enter the x-industry, less risk averse agents enter the y-industry

risk averse y-producers actually buy additional units of y in the futures market, whereas the least risk averse ones go short. Conventionally, the former would be considered a long speculative position, the latter would be considered a short hedge. The puzzle is, why would more risk-averse y-producers speculate while less risk-averse ones hedge? The answer is found in the effect income and price components of risk have on y-producers' expected utility. When the aggregate output is in the high state, y-producers sell their product at a relatively low price. They are, however, benefiting from the low price as consumers of y. On the other hand, when the aggregate output is low and y-producers can sell their product at a high price, with probability 0.5 their crop is almost completely destroyed by the low idiosyncratic shock. In this dreadful scenario, there is nothing to sell and there is no income to buy expensive y for y-producer's own consumption; the y-producer is starving. This is why the state with low aggregate output is the one risk-averse y-producers must insure against. They do so by taking long positions, that is, by "speculating". The least risk-averse y-producers do just the opposite. They take short positions that are profitable when the relative price is low. They transfer income to the "good" state, making the "bad" state even worse. They are comparatively indifferent to the possibility of a catastrophic outcome, if they are adequately compensated in more favorable circumstances. This explains the puzzle.

3.3 Industry Formation With A Binding No-Default Constraint

Proposition 6 showed that, with a non-binding no-default constraint, there is a single agent a' who is indifferent between the two industries, while the agents who are more risk-averse than a' become x-producers and the agents who are less risk-averse than a' become y-producers. This pattern of the industry formation changes dramatically when agents with greatly dispersed risk aversion, some almost risk-neutral, are exposed to severe production shocks, triggering the binding no-default constraint.

Figure 10 shows an example of the economy exposed to extremely severe aggregate and idiosyncratic shocks.Good y is produced with a technology that has a mean $\mu = 1$,

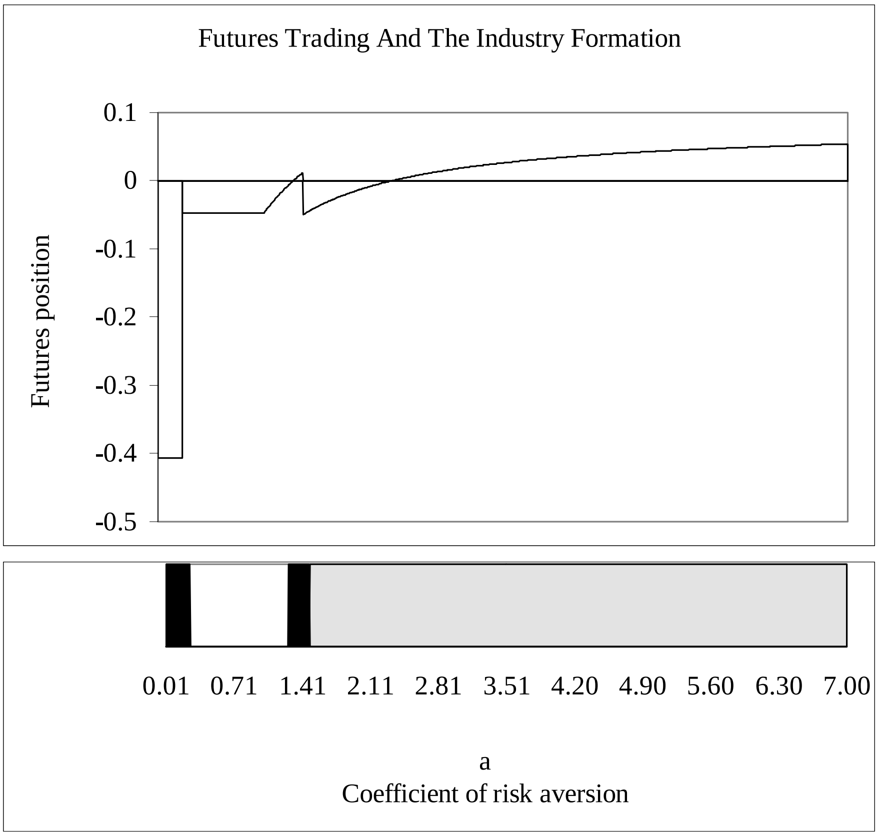


Figure 10: With large aggregate and idiosyncratic shocks,the no-default constraint binds, and speculators arise endogenously among the least risk averse agents.

the aggregate shock $\lambda = 0.8$ and the idiosyncratic shock $\omega = 0.18$. The population is uniformly distributed on the interval $(\mu_a - \lambda_a, \mu_a + \lambda_a) = (0.01, 7)$. In the bottom diagram, the shaded area again identifies the x-producers and the clear area shows the y-producers. The dark areas show agents who decide to switch the industry when they start trading futures. The most risk averse agents always become x-producers. Less risk averse agents become y-producers and many of them hedge by shorting the commodity to the maximum degree permitted by their worst case income as y-producers (the flat portion of $z(a)$ between $0.3 < a' < 1.2$). In stark contrast to the Proposition 6, the least risk-averse agents, $0.01 < a < 0.3$, switch industries and become *x-producers (!!)* when the futures market is available.

The reason for their industry choice is the severe no-default constraint that the idiosyncratic shock places on the short positions of y-producers. The least risk averse agents choose to enter the x-industry so they can take extreme short positions and increase their risk exposure. The dramatic difference between the short positions taken by y-producers and the short positions taken by the least risk averse x-producers is evident in the diagram. Those agents who switch to the x-industry in order to place large gambles in the futures market are the endogenous speculators.

The defection of the least risk-averse agents to the x-industry induces some moderately risk-averse x-producers, $1.3 < a < 1.5$, to switch to the y-industry. In this example, the measure of agents who become speculators slightly exceeds the measure of agents who switch to the y-industry. The net effect of the futures market is therefore an increased proportion of x-producers, which increases the price of good y. In Section 4, where we study changes in welfare of agents resulting from the introduction of the futures market, endogenous speculators turn out to benefit the most.

Futures Price and Expected Spot Price

In the previous section we showed that the futures price is not an unbiased estimator of the expected spot price in the absence of the idiosyncratic shock. The presence of backwardation or contango is tied to the distribution of the random output of y, and to the risk aversion of the population.

The effect of the idiosyncratic shock on backwardation and contango is represented in the following two diagrams. They show the ratio $p_F/E(p)$ as a function of the aggregate shock for three different values of the average coefficient of risk aversion. When backwardation occurs, $p_F/E(p) < 1$, while $p_F/E(p) > 1$ shows contango. They are based on the same parameter values as Figures 2 and 4 in the previous section: the values $\mu = 1$ and $\lambda_a = 1$ are the same in both diagrams. In Figure 11, the average risk aversion, $\mu_a = 2$, is weak, and Figure 12 shows strong average risk aversion, $\mu_a = 4$. The ratio of the futures price and the expected spot price, when y-producers are exposed to the idiosyncratic shock $\omega = 0.2$, is graphed as a thick curve. For comparison, the thin curve shows the ratio of the futures price and the expected spot price with no idiosyncratic shock. In both examples the ratio becomes greater on account of the idiosyncratic shock, although the effect is slight. Since the idiosyncratic shock adds risk to the y-producers' income, they tend to reduce their risk exposure by taking larger long positions or smaller short positions. To clear the future market, the futures price must be relatively higher. As a result, the ratio of the futures price and the expected spot price is also higher.

The logic of the relationship between the average risk aversion μ_a and the backwardation and contango remains the same as in the previous section. Equilibrium futures price is a weighted average of $\bar{p}$ and $\underline{p}$. If the population consists of strongly risk averse individuals (μ_a is relatively large) who tend to reduce their risk in utility by taking larger long positions, the futures market clears at a futures price that is relatively high, close to $\bar{p}$. This results in contango, the upward bias of the futures price $p_F > E(p)$. On the other hand, if the population consists of less risk averse agents (μ_a is relatively small) who tend to increase their risk in utility by taking large short positions, the equilibrium is achieved at a relatively low futures price, close to $\underline{p}$. The futures price is biased downward and backwardation $p_F < E(p)$ is observed.

4 Welfare Effects Of A Futures Market

Equivalent variation is used to measure the effect of the futures market on agents' welfare. A convenient way to introduce equivalent variation is the expansion of the model into two periods.

Consider a minor change in the model we studied in Sections 2 and 3. Suppose that we insert a period 0, a sort of a "prequel", in which production and consumption are not stochastic. In this period 0, agents choose the industry they want to enter, they produce, generate income and consume their preferred bundle. The output produced in period 0 cannot be stored and agents cannot save their income.

An agent who chooses the x-industry produces one unit of x and, since x is the numeraire, his income is 1. An agent who chooses the y-industry produces μ units of y and his income is $p_0\mu$. If each agent spends a proportion β of his income on the good x, then the period 0 spot market clears when the quantity demanded equals the quantity supplied

$$\beta A + \beta(1 - A) p_0\mu = A_0.$$

The market clearing spot price in period 0 is then

$$p_0 = \frac{(1-\beta)}{\beta}\frac{A_0}{(1-A_0)}\frac{1}{\mu}.$$

In the first period equilibrium, agents have the same income regardless of the industry they enter: $p_0\mu = 1$. That implies $A_0 = \beta$, and β agents enter the x-industry and $(1-\beta)$ agents enter the y-industry. The equilibrium spot price is then

$$p_0 = \frac{1}{\mu}.$$

The behavior of agents in period 1 is as described in Sections 2 and 3. For simplicity, agents can choose to enter a different industry each period. Agents' industry choice in period 0 has therefore no connection to their industry choice in period 1. That is why extending the model to two periods does not alter any of the results that were obtained thus far.

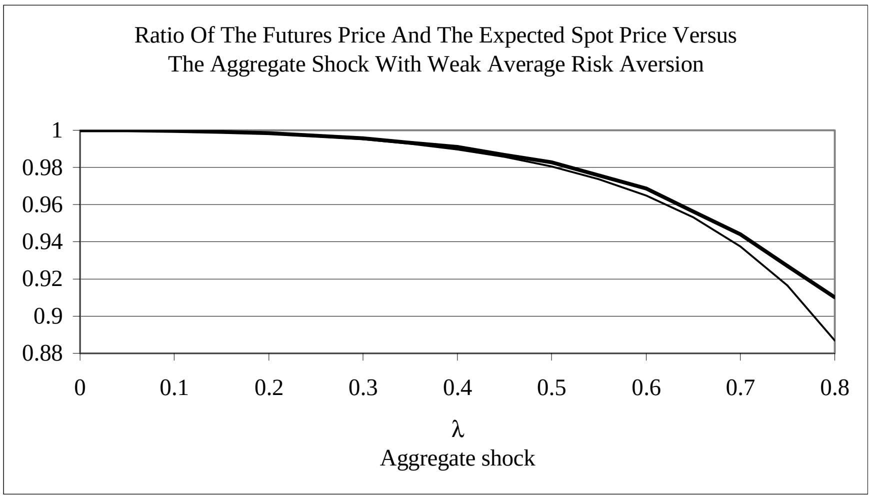


Figure 11: Weak average risk aversion ($\mu_a = 2$) results in backwardation for all λ

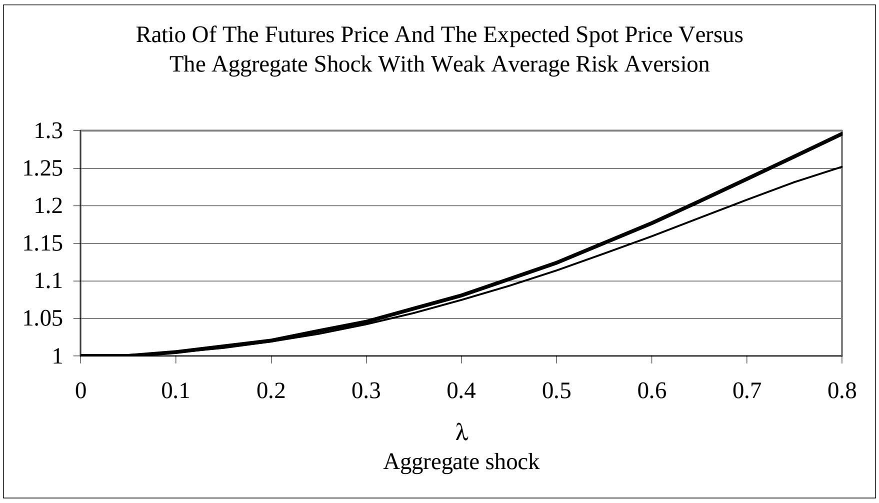


Figure 12: Strong average risk aversion ($\mu_a = 4$) results in contango for all λ

4.1 Welfare Effect When $\omega = 0$

With period 0, the utility functions (1.a) and (6.a) become

$$\begin{aligned} EV_{NF} &= -\exp\left[-a\frac{\widehat{\beta}}{p_0^{1-\beta}}\right] - \delta\left\{\frac{1}{2}\exp\left[-a\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\right] + \frac{1}{2}\exp\left[-a\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\right]\right\}, \text{ and} \\ EV_F &= -\exp\left[-a\frac{\widehat{\beta}}{p_0^{1-\beta}}\right] - 2\delta M K^{1+M}\exp\left[-a\frac{\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\left(\overline{p}-\underline{p}\right)}{\overline{D}\frac{\widehat{\beta}}{\overline{p}^{1-\beta}} + \underline{D}\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}}\right], \end{aligned}$$

where EV_F is the expected utility of an agent who has access to the futures market, EV_{NF} is the expected utility of the same agent without the access to the futures market, and δ is the discount rate.

In absence of the futures market, we measure the amount of good x each agent would require in excess of his income in period 0 in order to achieve the same level of expected utility he would enjoy with the futures market. When the aggregate shock is the only source of risk ($\omega = 0$), the equivalent variation VAR is implicitly defined by

$$\begin{aligned} EV_{NF} &= -\exp\left[-a\frac{\widehat{\beta}}{p_0^{1-\beta}}(1+VAR)\right] - \delta\left\{\frac{1}{2}\exp\left[-a\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\right] + \frac{1}{2}\exp\left[-a\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\right]\right\} \\ &\equiv -\exp\left[-a\frac{\widehat{\beta}}{p_0^{1-\beta}}\right] - 2\delta M K^{1+M}\exp\left[-a\frac{\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\left(\overline{p}-\underline{p}\right)}{\overline{D}\frac{\widehat{\beta}}{\overline{p}^{1-\beta}} + \underline{D}\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}}\right] = EV_F. \end{aligned}$$

Figure 13 shows an example of futures trading and welfare changes in the economy populated by agents on the interval $(\mu_a - \lambda_a, \mu_a + \lambda_a) = (1, 4)$. In the top diagram, the coefficient of risk aversion is on the horizontal axis, while the futures position each agent takes is measured on the vertical axis. Positive values represent long positions and negative values are short positions. The bottom diagram measures the equivalent variation on the vertical axis. The horizontal axis is common for both diagrams. Good y is produced with a technology that has a mean $\mu = 1$ and the aggregate shock $\lambda = 0.6$. Agents spend proportion $\beta = 0.5$ on good x.

As results of Section 2 show, in the absence of the idiosyncratic output shock the futures market has no effect on the industry formation and the equilibrium proportion of x-producers is always $A = \beta$. It means that the distribution of the random spot price of the commodity remains the same whether the futures market is available or not. This implies that all agents, given the opportunity to trade futures, must be at least as well off as they are without it. In fact, as Figure 13 shows, all agents, except $\hat{a}$, who chooses $z = 0$, are strictly better off.

Figure 14 shows an example with the population $(\mu_a - \lambda_a, \mu_a + \lambda_a) = (0.1, 4.9)$, and all remaining parameters the same as in Figure 13. The pattern of futures trading in the top diagram indicates that the no-default constraint is binding.

The increase in utility is stronger in the economy that is populated by individuals who are more diverse in their attitude towards risk. In Figure 13, where the population is $(\mu_a - \lambda_a, \mu_a + \lambda_a) = (1, 4)$, the welfare improvement agents gain due to trading futures

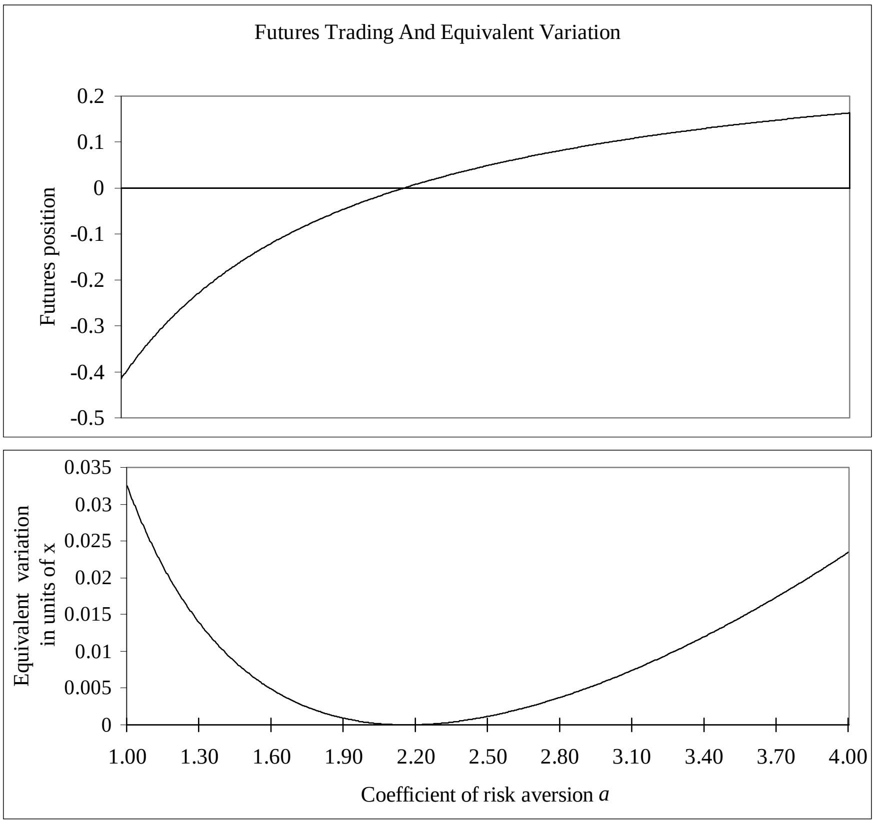


Figure 13: In absence of idiosyncratic shock, almost all agents are better off and no agent is worse off due to the futures market

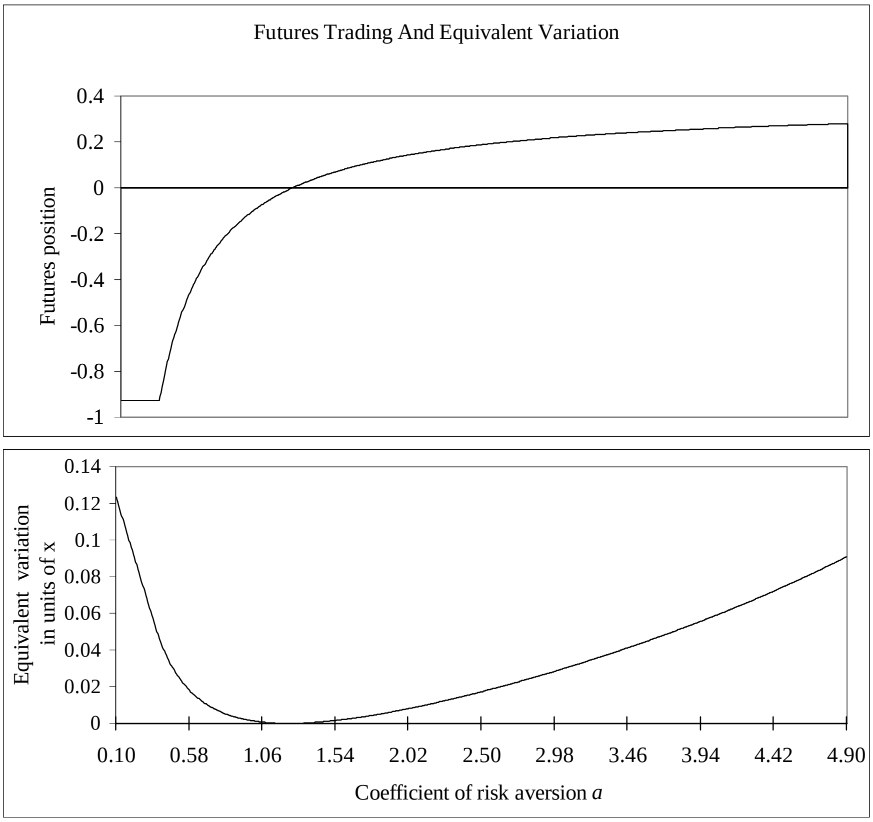


Figure 14: Increasing dispersion of the coefficient of risk aversion results in a higher equivalent variation of the least risk-averse and the most risk-averse agents

is valued at equivalent variation smaller than 0.035 units of good x. In Figure 14, with the population interval $(\mu_a - \lambda_a, \mu_a + \lambda_a) = (0.1, 4.9)$, the equivalent variation of some agents exceeds 0.1 units of good x, or 10% of their income in period 0.

4.2 Welfare Effects When $\omega > 0$

In the absence of the idiosyncratic production shock, introduction of the futures market has no effect on industry formation because the risk exposure in both industries is identical. Since the distribution of the relative spot price does not change, no agent's expected utility is decreased by the futures market. In fact, almost all agents benefit from the opportunity to trade futures.

The situation is radically different when y-producers are exposed to the combination of idiosyncratic and aggregate shocks. As the following examples demonstrate, there exists a nontrivial subset of agents whose expected utility is lower after the futures market is introduced.

As in the previous section, equivalent variation is used to measure the effect of the futures market on agents' welfare. In absence of the futures market, we measure the amount VAR of good x each agent would require in excess of his income in period 0 in order to achieve the same level of expected utility he would enjoy with the futures market.

Figure 15 shows the pattern of futures trading, welfare changes and industry formation in the economy populated by agents on the interval $(\mu_a - \lambda_a, \mu_a + \lambda_a) = (0.5, 7)$. In the top diagram, the population is on the horizontal axis, while the futures position each agent takes is measured on the vertical axis. Positive values represent long positions and negative values are short positions. The middle diagram measures the equivalent variation on the vertical axis. The bottom diagram has the population interval on the horizontal axis. The shaded area shows the x-producers and the clear area shows the y-producers. The narrow dark band dividing the x-producers from the y-producers is the subset of agents who switch from the x-industry to the y-industry when they trade futures. Good y is produced with a technology that has a mean $\mu = 1$, the aggregate shock $\lambda = 0.5$ and the idiosyncratic shock $\omega = 0.4$. Agents spend proportion $\beta = 0.5$ on good x.

For some y-producers, the value of equivalent variation is negative. These are the agents who are adversely affected by the futures market. The intuition behind this result follows from the fact that in this example the presence of the futures market increases the proportion of y-producers. The futures market offers an opportunity to manage risk and attracts more agents into the y-industry (the narrow dark region in the bottom diagram). The resulting increased supply of y drives the relative price of y, and y-producers' income, down. Those y-producers who choose not to trade futures, or trade very little, are hurt the most and their expected utility decreases.

Figure 16 shows an example of the economy exposed to extremely severe aggregate and idiosyncratic shocks. A binding no-default constraint has given rise to endogenous speculators.Good y is produced with a technology that has a mean $\mu = 1$, the aggregate shock $\lambda = 0.8$ and the idiosyncratic shock $\omega = 0.18$. The population is uniformly distributed on the interval $(\mu_a - \lambda_a, \mu_a + \lambda_a) = (0.01, 7)$. Agents spend proportion $\beta = 0.8$ on good x. In this example, the endogenous speculators benefit from the presence of the futures market the most. Recall that the period 0 income of all agents is 1. Figure 16

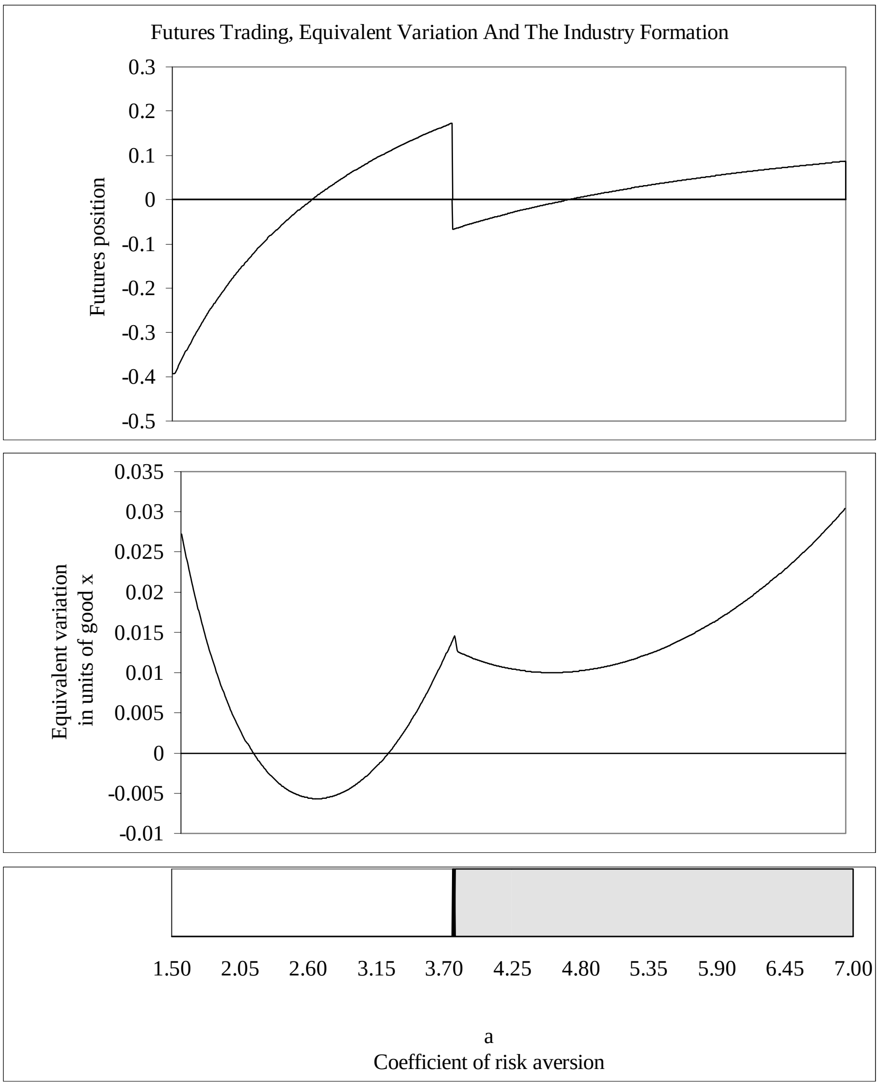


Figure 15: Futures market decreases expected utility of commodity producers who trade little

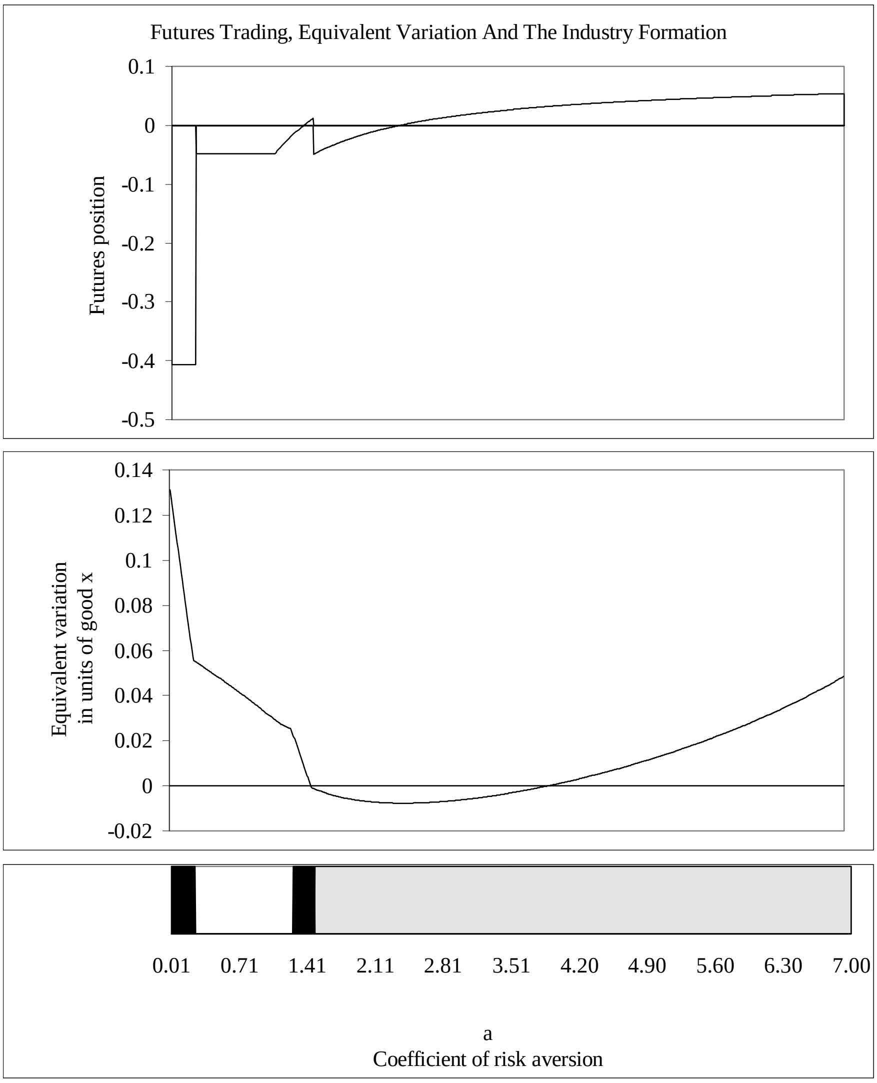


Figure 16: The futures market increases the expected utility of endogenous speculators the most

shows that the speculators value the presence of the futures market with the equivalent variation ranging between 0.06 and 0.12 units of x. In other words, the speculators' expected utility gain from futures trading is equivalent to 6 to 12 percent of their income in period 0. On the other hand, the x-producers who trade little are worse off because the relative price of y is higher with the futures market.

4.3 Futures Markets Versus Complete Markets: The Welfare Approach

In Section 2 of this paper we showed that the futures market has no impact on industry formation when the aggregate shock is the only source of uncertainty. Since risk can be completely shared through futures trading, markets are complete. In Section 3, we introduced the idiosyncratic shock. More risk-averse agents are no longer willing to enter the y-industry because futures trading does not help alleviate the effect of increased risk exposure. In the presence of the idiosyncratic shock, trading takes place in the framework of incomplete markets.

If the futures market was replaced by a complete set of contingent claims markets, the agents would be able to share the risk associated with the aggregate as well as idiosyncratic shocks. That would result in an equilibrium equivalent to the one with no idiosyncratic shock. We studied this equilibrium in Section 2. Comparison of equilibrium properties of the model with the aggregate shock on the one hand and the model with both the aggregate and idiosyncratic shocks on the other hand is equivalent to the comparison of complete contingent claims markets in contrast to the futures market.

We use the equivalent variation to measure the effect an introduction of complete contingent markets would have on utility of agents who can only trade futures. In Figure 17, equivalent variation is measured as the number of units of good x an agent would be willing to give up in period 0 in order to have access to a complete set of contingent claims instead of futures. This example is generated with the same parameter values as Figure 16: good y is produced with a technology that has a mean $\mu = 1$, the aggregate shock $\lambda = 0.8$ and the idiosyncratic shock $\omega = 0.18$. The population is distributed on the interval $(\mu_a - \lambda_a, \mu_a + \lambda_a) = (0.01, 7)$, and agents spend proportion $\beta = 0.8$ of their income on good x.

More risk averse agents value contingent claims positively, and equivalent variation is increasing with their degree of risk aversion. In stark contrast, most of the y-producers and the speculators would be harmed by the contingent claims. Speculators are worse off with contingent claims because they cannot profit by speculation anymore and the best they can do is to enter the y-industry. The y-producers, however, are disadvantaged by the inflow of competition drawn into the industry by the contingent markets. Notice that in Figure 16 the introduction of the futures market reduced the measure of the y-producers (the least risk-averse ones switched to the x-industry and became speculators), resulting in the higher relative price and higher utility of y-producers. With contingent claims, this advantage disappears and less risk-averse y-producers are worse off.

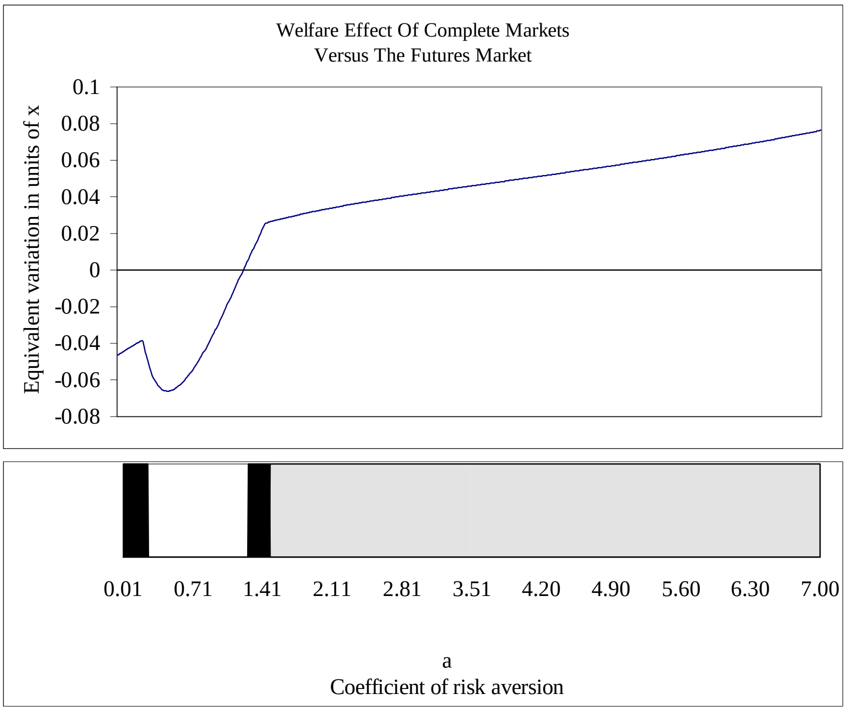


Figure 17: The presence of complete contingent claims markets benefits the more risk-averse agents, but is detrimental to the less risk-averse agents.

5 Conclusion

This paper shows general equilibrium effects of a futures market in a very simple model. Futures trading arises from the differential risk aversion of agents and a differential risk exposure in two industries. The combination of the income risk and the price risk gives rise to surprising patterns of futures trading that shed new light on the meaning of hedging and speculation. Both normal backwardation and contango arise in the model and are linked to the risk aversion of agents. The welfare analysis shows that not all agents benefit from the futures market. The general equilibrium effects of the futures market proved detrimental to some agents.

Appendix A

Proof of Proposition 4

A) Note that the two expected utility functions (6.a) and (6.b) differ by their last terms only. If the last term of EV^x is greater than that of EV^y, then each agent, regardless of his risk aversion, prefers the x-industry and vice versa. A situation in which all agents choose the same industry is not an equilibrium because markets for x and y do not clear. Therefore, in equilibrium, the two terms must be equal to each other. This implies $\frac{(1-\beta)}{\beta} = \frac{A}{(1-A)}$ or $A = \beta$. If $A < (>)\beta$, all agents would want to enter the x-industry (y-industry). Corresponding equilibrium price in two output states is obtained by substituting $A = \beta$ in (2).

B) Substituting $\bar{p} = \frac{1}{(\mu-\lambda)}$ and $\underline{p} = \frac{1}{(\mu+\lambda)}$ into z^y gives

$$\begin{aligned} z^y &= \frac{\left(\widehat{\beta}\underline{p}^{\beta}(\mu+\lambda) - \widehat{\beta}\overline{p}^{\beta}(\mu-\lambda)\right) - \frac{1}{a}\ln K}{\frac{\widehat{\beta}}{\bar{p}^{1-\beta}}\overline{D} + \frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\underline{D}} \\ &= \frac{\left(\frac{\widehat{\beta}}{\underline{p}^{1-\beta}} - \frac{\widehat{\beta}}{\bar{p}^{1-\beta}}\right) - \frac{1}{a}\ln K}{\frac{\widehat{\beta}}{\bar{p}^{1-\beta}}\overline{D} + \frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\underline{D}} \\ &= z^x \end{aligned}$$

Notice that $K > 1$ in equilibrium. If $K \leq 1$, all agents choose $z > 0$ and the futures market does not clear. With $K > 1$, futures position z is an increasing and concave function of a, as depicted in Figure 1. Setting

$$z = \frac{\left(\frac{\widehat{\beta}}{\underline{p}^{1-\beta}} - \frac{\widehat{\beta}}{\bar{p}^{1-\beta}}\right) - \frac{1}{a}\ln K}{\frac{\widehat{\beta}}{\bar{p}^{1-\beta}}\overline{D} + \frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\underline{D}} = 0$$

gives $\hat{a} = \frac{2\lambda_a}{\ln(\mu_a+\lambda_a)-\ln(\mu_a-\lambda_a)}$, the agent who chooses not to trade futures.

C) The futures market clearing condition $\int_{\mu_a-\lambda_a}^{\mu_a+\lambda_a} z\,da = 0$ gives

$$\begin{aligned} &\left(\widehat{\beta}(\mu+\lambda)^{1-\beta} - \widehat{\beta}(\mu-\lambda)^{1-\beta}\right) \\ = \;&(\ln(\mu_a+\lambda_a) - \ln(\mu_a-\lambda_a))\ln\left[\frac{(p_F-\underline{p})}{(\bar{p}-p_F)}\left(\frac{\bar{p}}{\underline{p}}\right)^{1-\beta}\right]. \end{aligned}$$

This yields equilibrium futures price p_F.

D) To simplify exposition, let's write the futures price as a weighted average of $\bar{p}$ and $\underline{p}$,

$$p_F = \frac{1}{1+\gamma}\bar{p} + \frac{\gamma}{1+\gamma}\underline{p},$$

where $\bar{p} = \frac{1}{(\mu-\lambda)}$, $\underline{p} = \frac{1}{(\mu+\lambda)}$ and

$$\gamma \equiv \left(\frac{\mu+\lambda}{\mu-\lambda}\right)^{1-\beta} \exp\left[-\frac{2\lambda_a\widehat{\beta}\left((\mu+\lambda)^{1-\beta}-(\mu-\lambda)^{1-\beta}\right)}{ln\left(\mu_a+\lambda_a\right)-\ln\left(\mu_a-\lambda_a\right)}\right].$$

Then the following inequalities hold:

$$\begin{aligned}
\frac{dp_F}{d\mu} &= -\left(\frac{1}{1+\gamma}\bar{p}^2+\frac{\gamma}{1+\gamma}\underline{p}^2\right) \\
&\quad -\frac{\gamma(1-\beta)\left(\bar{p}-\underline{p}\right)}{(1+\gamma)^2}\left[\frac{2\lambda_a\widehat{\beta}\left(\bar{p}^\beta-\underline{p}^\beta\right)}{ln\left(\mu_a+\lambda_a\right)-\ln\left(\mu_a-\lambda_a\right)}-\left(\bar{p}-\underline{p}\right)\right] \\
&< -\left[\left(\frac{1}{1+\gamma}\bar{p}^2+\frac{\gamma}{1+\gamma}\underline{p}^2\right)-\frac{\gamma}{(1+\gamma)^2}(1-\beta)\left(\bar{p}-\underline{p}\right)^2\right] \\
&< -\left[\left(\frac{1}{1+\gamma}\bar{p}^2+\frac{\gamma}{1+\gamma}\underline{p}^2\right)-\frac{\gamma}{(1+\gamma)^2}\left(\bar{p}-\underline{p}\right)^2\right] \\
&= -\left(\frac{\gamma}{1+\gamma}\right)^2\left(\frac{1}{\gamma}\bar{p}+\underline{p}\right)^2 \\
&< 0.
\end{aligned}$$

$$\begin{aligned}
\frac{dp_F}{d\lambda} &= \left(\frac{1}{1+\gamma}\bar{p}^2-\frac{\gamma}{1+\gamma}\underline{p}^2\right) \\
&\quad +\frac{\gamma(1-\beta)\left(\bar{p}-\underline{p}\right)}{(1+\gamma)^2}\left[\frac{2\lambda_a\widehat{\beta}\left(\bar{p}^\beta-\underline{p}^\beta\right)}{ln\left(\mu_a+\lambda_a\right)-\ln\left(\mu_a-\lambda_a\right)}-\left(\bar{p}-\underline{p}\right)\right] \\
&> \frac{1}{1+\gamma}\bar{p}^2-\frac{\gamma}{1+\gamma}\underline{p}^2-\frac{\gamma}{(1+\gamma)^2}(1-\beta)\left(\bar{p}^2+\underline{p}^2\right) \\
&> \frac{1}{1+\gamma}\bar{p}^2-\frac{\gamma}{1+\gamma}\underline{p}^2-\frac{\gamma}{(1+\gamma)^2}\left(\bar{p}^2+\underline{p}^2\right) \\
&= \frac{1}{(1+\gamma)^2}\left(\bar{p}+\gamma\underline{p}\right)\left(\bar{p}-\gamma\underline{p}\right) \\
&> 0.
\end{aligned}$$

The last inequality holds because $\bar{p} > \gamma\underline{p}$:

$$\begin{aligned}
\frac{1}{\mu-\lambda} &> \frac{1}{\mu+\lambda}\left(\frac{\mu+\lambda}{\mu-\lambda}\right)^{1-\beta}\exp\left[-\frac{2\lambda_a\widehat{\beta}\left((\mu+\lambda)^{1-\beta}-(\mu-\lambda)^{1-\beta}\right)}{ln\left(\mu_a+\lambda_a\right)-\ln\left(\mu_a-\lambda_a\right)}\right] \\
\left(\frac{\mu+\lambda}{\mu-\lambda}\right)^{\beta} &> \exp\left[-\frac{2\lambda_a\widehat{\beta}\left((\mu+\lambda)^{1-\beta}-(\mu-\lambda)^{1-\beta}\right)}{ln\left(\mu_a+\lambda_a\right)-\ln\left(\mu_a-\lambda_a\right)}\right],
\end{aligned}$$

which holds because

$$\left(\frac{\mu+\lambda}{\mu-\lambda}\right)^{\beta} > 1 > \exp\left[-\frac{2\lambda_a\widehat{\beta}\left((\mu+\lambda)^{1-\beta}-(\mu-\lambda)^{1-\beta}\right)}{ln\left(\mu_a+\lambda_a\right)-\ln\left(\mu_a-\lambda_a\right)}\right].$$

Finally, p_F is increasing in μ_a:

$$\frac{dp_F}{d\mu_a} = \frac{\left(\frac{1}{\mu-\lambda}-\frac{1}{\mu+\lambda}\right)}{(1+\gamma)^2}\frac{4\lambda_a\left[(\mu+\lambda)^{1-\beta}-(\mu-\lambda)^{1-\beta}\right]}{\left(\mu_a^2-\lambda_a^2\right)\ln\left(\frac{\mu_a+\lambda_a}{\mu_a-\lambda_a}\right)} > 0.$$

E) Since $p_F = \frac{1}{1+\gamma}\bar{p} + \frac{\gamma}{1+\gamma}\underline{p}$, the futures price is equal to the expected spot price if $\frac{1}{1+\gamma}\bar{p} + \frac{\gamma}{1+\gamma}\underline{p} = \frac{1}{2}\bar{p} + \frac{1}{2}\underline{p}$, or $\gamma = 1$. This is equivalent to

$$\frac{2\lambda_a}{\ln\left(\mu_a+\lambda_a\right)-\ln\left(\mu_a-\lambda_a\right)} = \frac{\ln\left(\mu+\lambda\right)^{1-\beta}-\ln\left(\mu-\lambda\right)^{1-\beta}}{\widehat{\beta}\left(\mu+\lambda\right)^{1-\beta}-\widehat{\beta}\left(\mu-\lambda\right)^{1-\beta}}.$$

Q.E.D.

Appendix B

Proof of Proposition 5

A) The proof proceeds in two steps. First, we will show that there exists an agent a' who is indifferent between the two industries, if all agents $a \in (\mu_a - \lambda_a, a')$ enter y-industry and all $a \in (a', \mu_a + \lambda_a)$ enter x-industry. In the second step, we will show that this pattern of industry formation actually arises because agents choose the enter respective industries to maximize their expected utility, and that $A > \beta$.

Step 1

Let's consider an agent $a' \in (\mu_a - \lambda_a, \mu_a + \lambda_a)$ and assume that all agents $a \in (\mu_a - \lambda_a, a')$ enter the y-industry and all $a \in (a', \mu_a + \lambda_a)$ enter the x-industry. If $a' \to \mu_a - \lambda_a$, almost all agents are x-producers, while almost no one produces y. The relative price p approaches infinity, which makes $EV^y \to 0$ and $EV^x \to -1$ for all agents. It means that if $a' \to \mu_a - \lambda_a$, then $EV^x_{a'} < EV^y_{a'}$. Conversely, if $a' \to \mu_a + \lambda_a$, the relative price p approaches zero, which makes $EV^y \to -1$ and $EV^x \to 0$ for all agents. It means that if $a' \to \mu_a + \lambda_a$, then $EV^x_{a'} > EV^y_{a'}$. As a' increases from $\mu_a - \lambda_a$ towards $\mu_a + \lambda_a$, EV^y decreases while EV^x increases for every agent. Continuity of all functions then implies that there must exist an agent a' such that $EV^x_{a'} = EV^y_{a'}$.

Step 2

The x-producer's utility function is

$$EV^x = -\frac{1}{2}\exp\left[-a\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\right] - \frac{1}{2}\exp\left[-a\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\right].$$

Substitution of market clearing price (2) in the y-producers utility function yields

$$\begin{aligned} EV^y \quad = \quad & -\frac{1}{2}\underline{C}\exp\left[-a\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\frac{(1-\beta)}{\beta}\frac{A}{(1-A)}\right] \\ & -\frac{1}{2}\overline{C}\exp\left[-a\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\frac{(1-\beta)}{\beta}\frac{A}{(1-A)}\right]. \end{aligned}$$

To simplify the notation, let's set $\gamma \equiv \frac{1-\beta}{\beta}\frac{A}{1-A}$ and write the utility function of a y-producer as:

$$\begin{aligned} EV^y \quad = \quad & -\frac{1}{2}\exp\left[-a\left(\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\gamma - \frac{1}{a}\ln\underline{C}\right)\right] \\ & -\frac{1}{2}\exp\left[-a\left(\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\gamma - \frac{1}{a}\ln\overline{C}\right)\right]. \end{aligned}$$

Comparing EV^x and EV^y shows that if $\gamma = 1$, then $EV^x > EV^y$ for all agents, because $\underline{C}, \overline{C} > 1$ for all positive values of the idiosyncratic shock ω. Therefore, in equilibrium it must be the case that $\gamma > 1$, which is identical to $A > \beta$.

Notice that both the x-producers and the y-producer have a higher utility in the high output/low price of good y state of the world: $\frac{\widehat{\beta}}{\underline{p}^{1-\beta}} > \frac{\widehat{\beta}}{\overline{p}^{1-\beta}}$ and $\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\gamma - \frac{1}{a}\ln \underline{C} > \frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\gamma - \frac{1}{a}\ln \overline{C}$. The last inequality holds because hyperbolic cosine is an increasing function.

In equilibrium, it must be the case that the marginal agent who is indifferent between the two industries has

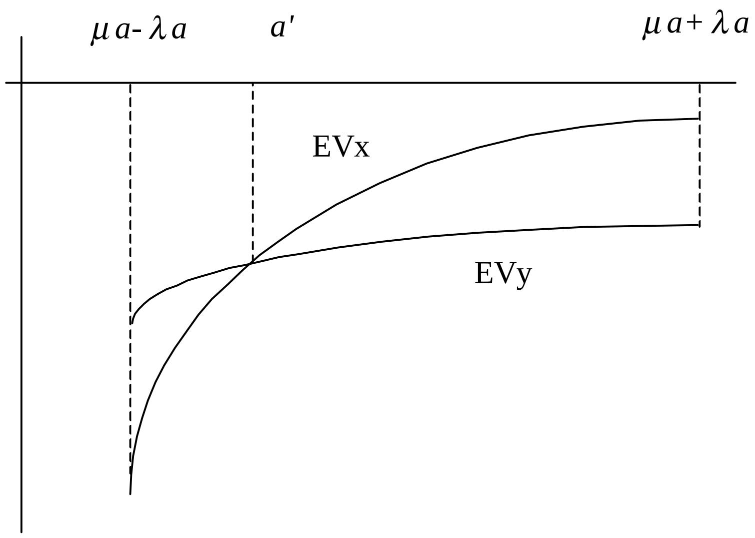
$$\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\gamma - \frac{1}{a}\ln \underline{C} > \frac{\widehat{\beta}}{\underline{p}^{1-\beta}} > \frac{\widehat{\beta}}{\overline{p}^{1-\beta}} > \frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\gamma - \frac{1}{a}\ln \overline{C}.$$

To show this, assume instead that $\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\gamma - \frac{1}{a}\ln \underline{C} = \frac{\widehat{\beta}}{\underline{p}^{1-\beta}}$. Then the following chain of inequalities holds:

$$\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}(\gamma - 1) < \frac{\widehat{\beta}}{\underline{p}^{1-\beta}}(\gamma - 1) = \frac{1}{a}\ln \underline{C} < \frac{1}{a}\ln \overline{C}.$$

This implies $\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\gamma - \frac{1}{a}\ln \overline{C} < \frac{\widehat{\beta}}{\overline{p}^{1-\beta}}$ and the agent would prefer the x-industry, which is a contradiction.

To prove that less risk averse agents $a < a'$ enter the y-industry and more risk averse agents $a > a'$ enter the x-industry, it is sufficient to show that $\frac{dEV^x}{da}\big|_{a=a'} > \frac{dEV^y}{da}\big|_{a=a'}$. See the diagram.



We obtain the result from the following chain of inequalities:

$$\begin{aligned}
\frac{dEV^y}{da}\Big|_{a=a'} &= \frac{1}{2}\left(\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\gamma - \frac{1}{a}\ln \underline{C}\right)\exp\left[-a\left(\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\gamma - \frac{1}{a}\ln \underline{C}\right)\right] \\
&\quad +\frac{1}{2}\left(\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\gamma - \frac{1}{a}\ln \overline{C}\right)\exp\left[-a\left(\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\gamma - \frac{1}{a}\ln \overline{C}\right)\right] \\
&\quad -a\frac{d}{da}\left(\frac{1}{a}\ln \underline{C} + \frac{1}{a}\ln \overline{C}\right) \\
&< \frac{1}{2}\left(\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\gamma - \frac{1}{a}\ln \underline{C}\right)\exp\left[-a\left(\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\gamma - \frac{1}{a}\ln \underline{C}\right)\right] \\
&\quad +\frac{1}{2}\left(\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\gamma - \frac{1}{a}\ln \overline{C}\right)\exp\left[-a\left(\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\gamma - \frac{1}{a}\ln \overline{C}\right)\right] \\
&< \frac{1}{2}\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\exp\left[-a\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\right] + \frac{1}{2}\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\exp\left[-a\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\right] \\
&= \frac{dEV^x}{da}\Big|_{a=a'} .
\end{aligned}$$

The first inequality holds because $\frac{d}{da}\left(\frac{1}{a}\ln \underline{C} + \frac{1}{a}\ln \overline{C}\right) > 0$.
To prove the second inequality, it is sufficient to show that

$$\begin{aligned}
&\frac{1}{2}a\left(\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\gamma - \frac{1}{a}\ln \underline{C}\right)\exp\left[-a\left(\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\gamma - \frac{1}{a}\ln \underline{C}\right)\right] \\
&+\frac{1}{2}a\left(\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\gamma - \frac{1}{a}\ln \overline{C}\right)\exp\left[-a\left(\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\gamma - \frac{1}{a}\ln \overline{C}\right)\right] \\
< \; &\frac{1}{2}a\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\exp\left[-a\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\right] + \frac{1}{2}a\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\exp\left[-a\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\right].
\end{aligned}$$

For convenience, let's define

$$\begin{aligned}
S &\equiv \exp\left[-a\left(\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\gamma - \frac{1}{a}\ln \underline{C}\right)\right], \\
T &\equiv \exp\left[-a\left(\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\gamma - \frac{1}{a}\ln \overline{C}\right)\right], \\
M &\equiv \exp\left[-a\frac{\widehat{\beta}}{\underline{p}^{1-\beta}}\right], \text{ and} \\
N &\equiv \exp\left[-a\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\right].
\end{aligned}$$

The inequality then changes to

$$\frac{1}{2}S\ln S+\frac{1}{2}T\ln T>\frac{1}{2}M\ln M+\frac{1}{2}N\ln N.$$

We also know that the marginal agent a' is indifferent between the two industries, $EV^x = EV^y$, or

$$\frac{1}{2}S+\frac{1}{2}T=\frac{1}{2}M+\frac{1}{2}N,$$

where $S > M > N > T$.

Let's use the last equation and define $\Delta \equiv S-M$. Then $N-T=\Delta$ and the inequality can be written as

$$\begin{aligned} & \frac{1}{2}(M+\Delta)\ln(M+\Delta)+\frac{1}{2}(N-\Delta)\ln(N-\Delta) \\ > \; & \frac{1}{2}M\ln M+\frac{1}{2}N\ln N. \end{aligned}$$

This must be true because the function

$$\frac{1}{2}(M+\Delta)\ln(M+\Delta)+\frac{1}{2}(N-\Delta)\ln(N-\Delta)$$

is increasing in Δ:

$$\begin{aligned} & \frac{d}{d\Delta}\left(\frac{1}{2}(M+\Delta)\ln(M+\Delta)+\frac{1}{2}(N-\Delta)\ln(N-\Delta)\right) \\ = \; & \frac{1}{2}\left(\ln(M+\Delta)-\ln(N-\Delta)\right) \\ = \; & \frac{1}{2}\left(\ln S-\ln T\right) \\ > \; & 0. \end{aligned}$$

B) Suppose that the agent a' is indifferent between the two industries, $EV^x_{a'} = EV^y_{a'}$. Let's consider an increase in the size of the idiosyncratic shock ω and keep other things, including the industry formation, constant. While EV^x is not affected, EV^y decreases because both $\underline{C}$ and $\overline{C}$ increase. As a result, $EV^x_{a'} > EV^y_{a'}$, agent a' enters the x-industry and A increases.

Let $a_1 < \mu_a$ be the marginal agent in a population $(\mu_a - \lambda_a, \mu_a + \lambda_a)$. If λ_a increases, the new marginal agent must be $a_2 < a_1$, so that the relative price is preserved. The agent a_2 is, however, less risk averse than a_1 and strictly prefers to enter the y-industry. Agent a_2 therefore enters the y-industry and A decreases.

Q.E.D.

Appendix C

Proof of Proposition 6

If $A = \beta$, $EV^x > EV^y$ for all agents, because $\underline{C}, \overline{C} > 1$. That is not consistent with equilibrium. Proportion of x-producers must therefore be $A > \beta$.

As in Proposition A.3, it is assumed that all agents $a \in (\mu_a - \lambda_a, a')$ enter the y-industry and all $a \in (a', \mu_a + \lambda_a)$ enter the x-industry. First, notice that for every agent there exists a futures price p'_F that makes him indifferent between the two industries. At the futures price p'_F, for any futures position z^x this agent would choose if he entered the x-industry, there exists a position z^y that gives this agent the same expected utility in both states of the world if he entered the y-industry:

$$\begin{aligned} \widehat{\beta}\underline{p}^{\beta}(\mu+\lambda) - \frac{1}{a}\ln\underline{C} - \underline{D}z^y &= \frac{\widehat{\beta}}{\underline{p}^{1-\beta}} - \underline{D}z^x, \text{ and} \\ \widehat{\beta}\overline{p}^{\beta}(\mu-\lambda) - \frac{1}{a}\ln\overline{C} + \overline{D}z^y &= \frac{\widehat{\beta}}{\overline{p}^{1-\beta}} + \overline{D}z^x. \end{aligned}$$

Solving these two equations simultaneously gives

$$\begin{aligned} p'_F = & \left[\frac{(\gamma-1)\frac{\widehat{\beta}}{\underline{p}^{1-\beta}} - \frac{1}{a}\ln\underline{C}}{(\gamma-1)\left(\frac{\widehat{\beta}}{\underline{p}^{1-\beta}} - \frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\right) - \frac{1}{a}\ln\left(\frac{\overline{C}}{\underline{C}}\right)}\right]\overline{p} \\ & + \left[\frac{\frac{1}{a}\ln\overline{C} - (\gamma-1)\frac{\widehat{\beta}}{\overline{p}^{1-\beta}}}{(\gamma-1)\left(\frac{\widehat{\beta}}{\underline{p}^{1-\beta}} - \frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\right) - \frac{1}{a}\ln\left(\frac{\overline{C}}{\underline{C}}\right)}\right]\underline{p} \end{aligned}$$

and

$$z^y = \frac{(\gamma-1)\left(\frac{\widehat{\beta}}{\underline{p}^{1-\beta}} - \frac{\widehat{\beta}}{\overline{p}^{1-\beta}}\right) - \frac{1}{a}\ln\left(\frac{\overline{C}}{\underline{C}}\right)}{\overline{p} - \underline{p}} + z^x,$$

where $\gamma \equiv \frac{1-\beta}{\beta}\frac{A}{1-A}$. In other words, at the futures price p'_F, a y-producer a' can take the futures position z^y and get the same expected utility he would get as an x-producer trading z^x.

A direct consequence of this fact is that any no-futures equilibrium A_{NF} can be supported by the futures price p'_F such that spot market clears. The problem is, however, that p'_F does not, in general, clear the futures market. That is why a simultaneous clearing of both the spot and the futures markets results in a different industry formation.

To show that there in fact exists equilibrium at which the marginal agent a' is indifferent between the two industries and the futures market clears, consider two extreme cases. First, there is an agent a', sufficiently close to the lower bound $\mu_a - \lambda_a$ of the population interval, such that $S > M$ and $T = N$, using notation introduced in the proof of Proposition 5. Notice that the futures price p'_F for this agent is equal to $\overline{p}$. This futures price obviously cannot clear the futures market because all agents take short positions, betting that the spot price will be lower than the futures price, and

$$\int_G z^x d\nu(a) + \int_H z^y d\nu(a) < 0.$$

Conversely, there is an agent a', sufficiently close to the upper bound $\mu_a + \lambda_a$ of the population interval, such that $S = M$ and $T < N$. The futures price p'_F for this agent is equal to $\underline{p}$. Now, however, all agents take long positions, betting that the spot price will be higher than the futures price, and

$$\int_G z^x d\nu(a) + \int_H z^y d\nu(a) > 0.$$

Because all functions are continuous, between the two extremes there must be a marginal agent a' such that

$$\int_G z^x d\nu(a) + \int_H z^y d\nu(a) = 0.$$

Now we can show that all agents $a < a'$ enter the y-industry and all agents $a > a'$ enter the x-industry. Consider an agent $a > a'$ who is more risk averse than a'. In order to be indifferent, this agent would require the futures price p'_F to be higher than the equilibrium futures price because $dp'_F/da > 0$. Working backwards, we see that if equilibrium futures price $p_F < p'_F$, then

$$\begin{aligned}
\widehat{\beta}\underline{p}^{\beta}(\mu + \lambda) - \frac{1}{a}\ln \underline{C} - \underline{D}z^y &< \frac{\widehat{\beta}}{\underline{p}^{1-\beta}} - \underline{D}z^x, \text{ and} \\
\widehat{\beta}\overline{p}^{\beta}(\mu - \lambda) - \frac{1}{a}\ln \overline{C} + \overline{D}z^y &< \frac{\widehat{\beta}}{\overline{p}^{1-\beta}} + \overline{D}z^x.
\end{aligned}$$

In words, for any futures position z^y the more risk averse agent $a > a'$ chooses in the y-industry, there exists a position z^x he can take in the x-industry and be better off in each state of the world. This agent therefore strictly prefers the x-industry. Analogous argument applies to less risk averse agents $a < a'$ who prefer to enter the y-industry.

Q.E.D.

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